

**Master Course in Mathematical Engineering — 2017/18**  
**Advanced Analysis 1– DT0114 – 6 CFU**  
**Lecturer: C. Lattanzio**

**Distributions.** Locally integrable functions. The space of test function  $\mathcal{D}(\Omega)$ . Distributions. Distributions associated to locally integrable functions. Singular distributions. Examples. Operations on distributions: sum, products times functions, change of variables, restrictions, tensor product. Differentiation and his properties; comparison with classical derivatives. Differentiation of jump functions. Partition of unity. Support of a distribution; compactly supported distributions.

**Convolution.** Convolution in  $L^p$  spaces. Regularity of the convolution. Regularizing sequences and smoothing by means of convolutions. Convolution between distributions and regularization of distributions. Denseness of  $\mathcal{D}(\Omega)$  in  $\mathcal{D}'(\Omega)$ .

**Sobolev spaces.** Definition of weak derivatives and his motivation. Sobolev spaces  $W^{k,p}(\Omega)$  and their properties. Interior and global approximation by smooth functions. Extensions. Traces. Embeddings theorems: Gagliardo–Nirenberg–Sobolev inequality and embedding theorem for  $p < n$ . Hölder spaces. Morrey inequality. Embedding theorem for  $p > n$ . Sobolev inequalities in the general case. Compact embeddings: Rellich–Kondrachov theorem, Poincaré inequalities. Embedding theorem for  $p = n$ . Characterization of the dual space  $H^{-1}$ .

**Second order parabolic equations.** Definition of parabolic operator. Weak solutions for linear parabolic equations. existence of weak solutions: Galerkin approximation, construction of approximating solutions, energy estimates, existence and uniqueness of solutions.

**First order nonlinear hyperbolic equations.** Scalar conservation laws: derivation, examples. Weak solutions, Rankine-Hugoniot conditions, entropy conditions.  $L^1$  stability, uniqueness and comparison for weak entropy solutions. Convergence of the vanishing viscosity and existence of the weak, entropy solution. Riemann problem.

**Textbooks:**

- H. Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations*. Universitext, Springer.
- C.M. Dafermos, *Hyperbolic Conservation Laws in Continuum Physics*, Springer.
- L.C. Evans, *Partial Differential Equations*. Graduate Studies in Mathematics, Vol. 19, AMS.
- G. Gilardi, *Analisi 3*. McGraw–Hill.
- V.S. Vladimirov, *Equations of Mathematical Physics*. Marcel Dekker, Inc.