2°) L.T.I. , T.D.

2° ESERCIZIO
15/02/2022

$$\phi(t) = \begin{bmatrix} 2^t & 0 \\ 2^t - (1/2)^t & (1/2)^t \end{bmatrix} = A^t \rightarrow A = \phi(1) = \begin{bmatrix} 2 & 0 \\ 3/2 & 1/2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -1 \end{bmatrix}, \quad D = \emptyset$$

A è una matrice triangolare inferiore quindi

$\lambda_1 = 2, \quad \lambda_2 = 1/2$

$|\lambda_1| = 2 > 1 \rightarrow$ il modo naturale associato è instabile

$|\lambda_2| = 1/2 < 1 \rightarrow$ il " " " è A.S.

$$v_1 : |\lambda_1 I - A| \cdot v_1 = 0 \leftrightarrow \begin{bmatrix} \lambda_1 - 2 & 0 \\ -3/2 & \lambda_1 - 1/2 \end{bmatrix} \cdot v_1 = 0 \leftrightarrow \begin{bmatrix} 0 & 0 \\ -3/2 & +3/2 \end{bmatrix} \cdot v_1 = 0$$

$$\rightarrow v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$v_2 : |\lambda_2 I - A| \cdot v_2 = 0 \leftrightarrow \begin{bmatrix} -3/2 & 0 \\ -3/2 & 0 \end{bmatrix} \cdot v_2 = 0 \rightarrow v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$R = \begin{bmatrix} v_1 & \vdots & v_2 \end{bmatrix} = \begin{bmatrix} 1 & \vdots & 0 \\ 1 & \vdots & 1 \end{bmatrix} \rightarrow L = R^{-1} = \frac{1}{1} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} e_1^T \\ e_2^T \end{bmatrix}$$

$$C \cdot v_1 = \begin{bmatrix} 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0 \rightarrow$$ il modo naturale associato all'autovalore λ_1 non è osservabile

$$C \cdot v_2 = \begin{bmatrix} 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -1 \neq 0 \rightarrow$$ il modo naturale associato all'autovalore λ_2 è osservabile

$$\left.\begin{aligned} e_1^T \cdot B &= \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1 \neq 0 \\ e_2^T \cdot B &= \begin{bmatrix} -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = -1 \neq 0 \end{aligned}\right\}$$ entrambi i modi naturali sono eccitabili

$$\bullet\ w(t) = C \cdot A^{t-1} \cdot B + \underset{0}{\underset{\parallel}{D}}\,\delta(t) =$$

$$= \begin{bmatrix} 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2^{t-1} & 0 \\ 2^{t-1} - \left(\frac{1}{2}\right)^{t-1} & \left(\frac{1}{2}\right)^{t-1} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} =$$

$$= 2^{t-1} - 2^{t-1} + \left(\frac{1}{2}\right)^{t-1} = \left(\frac{1}{2}\right)^{t-1}$$

$$\bullet\ W(z) = \mathcal{Z}(w(t)) = \frac{1}{z - 0.5}$$

$$\bullet\ Y_F(z) = W(z) \cdot U(z) \quad , u(t) = \delta_{-1}(t) \to U(z) = \frac{z}{z-1}$$

$$Y_F(z) = \frac{z}{(z-0.5)(z-1)} \iff \frac{Y_F(z)}{z} = \frac{R_1}{(z-0.5)} + \frac{R_2}{(z-1)}$$

$$R_1 = \lim_{z \to 0.5} \frac{1}{z-1} = -2 \quad , R_2 = \lim_{z \to 1} \frac{1}{z-0.5} = 2$$

$R_1 + R_2 = 0$ <u>OK</u>

$$\to Y_F(z) = -2 \frac{z}{z-0.5} + 2 \frac{z}{z-1} \Rightarrow y_F(t) = -2 \left(\frac{1}{2}\right)^t + 2\,\delta_{-1}(t)$$

$$\bullet\ u(t) = \sin\left(\frac{\pi}{2} t\right) \to U(z) = \frac{z \cdot \sin(\pi/2)}{z^2 - 2\cos(\pi/2) z + 1} = \frac{z}{z^2+1} = \frac{z}{(z+j)(z-j)}$$

$$\Rightarrow Y_F(z) = \frac{z}{(z-0.5)(z+j)(z-j)} \iff \frac{Y_F(z)}{z} = \frac{R_1}{z-0.5} + \frac{R_2}{z+j} + \frac{R_2^*}{z-j}$$

$$R_1 = \lim_{z \to 0.5} \frac{1}{(z^2+1)} = \frac{1}{\frac{1}{4}+1} = \frac{1}{\frac{5}{4}} = \frac{4}{5}$$

$$R_2 = \lim_{z \to -j} \frac{1}{(z-0.5)(z-j)} = \frac{1}{(+j+0.5)(+2j)} = \frac{j}{2} \cdot \frac{1}{(0.5+j)} = -\frac{j}{2} \cdot \frac{0.5-j}{\frac{1}{4}+1} = -\frac{2}{5}\left(1 + \frac{j}{2}\right)$$

$$R_2^* = \frac{2}{5}\left(-1 + \frac{j}{2}\right)$$

$$R_1 + R_2 + R_2^* = \frac{4}{5} + \frac{2}{5}\left(-1-\frac{j}{2}\right) + \frac{2}{5}\left(-1+\cancel{\frac{j}{2}}\right) = \emptyset \quad \text{OK}$$

$$Y_F(z) = \frac{4}{5}\,\frac{z}{z-0.5} - \frac{2}{5}\left(1+\frac{j}{2}\right)\frac{z}{z+j} + \frac{2}{5}\left(-1+\frac{j}{2}\right)\frac{z}{z-j}$$

$$y_F(t) = \frac{4}{5}\left(\frac{1}{2}\right)^t - \frac{2}{5}\left(1+\frac{j}{2}\right)(-j)^t + \frac{2}{5}\left(-1+\frac{j}{2}\right)(j)^t$$

$$j = e^{j\frac{\pi}{2}} \rightarrow -\frac{2}{5}\left(e^{-j\frac{\pi}{2}t} + e^{j\frac{\pi}{2}t}\right) - \frac{j}{5}\left(e^{-j\frac{\pi}{2}t} - e^{j\frac{\pi}{2}t}\right)\cdot\frac{2}{2} =$$

$$-j = e^{-j\frac{\pi}{2}}$$

$$= -\frac{4}{5}\cdot\cos\left(\frac{\pi}{2}t\right) - \frac{2}{5}\sin\left(\frac{\pi}{2}t\right)$$

$$\rightarrow y_F(t) = \frac{4}{5}\left(\frac{1}{2}\right)^t - \frac{4}{5}\cos\left(\frac{\pi}{2}t\right) - \frac{2}{5}\sin\left(\frac{\pi}{2}t\right)$$

3°) T.C. $\quad p = q = 1$

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$$W(s) = \frac{4}{(s+1)(s+2)}$$

- $$w(t) = \mathcal{L}^{-1}(W(s)) = \mathcal{L}^{-1}\left(\frac{R_1}{(s+1)} + \frac{R_2}{(s+2)}\right) = \otimes$$

$$R_1 = \lim_{s\to -1}\frac{4}{s+2} = 4 \quad , \quad R_2 = \lim_{s\to -2}\frac{4}{s+1} = -4$$

$$\otimes = 4\,\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) - 4\,\mathcal{L}^{-1}\left(\frac{1}{s+2}\right) = 4\cdot e^{-t} - 4\,e^{-2t}$$

- $$U(s) = \mathcal{L}(\delta_{-1}(t)) = \frac{1}{s} \Rightarrow Y_F(s) = W(s)\,U(s) = \frac{4}{(s+1)(s+2)}\cdot\frac{1}{s} =$$

$$= \frac{R_1}{s+1} + \frac{R_2}{s+2} + \frac{R_3}{s} = \otimes$$

$$R_1 = \lim_{s\to -1}\frac{4}{(s+2)\cdot s} = -4$$

$$R_2 = \lim_{s\to -2}\frac{4}{(s+1)(s)} = 2$$

$$R_3 = \lim_{s\to 0}\frac{4}{(s+1)(s+2)} = \frac{4}{2} = 2$$

$R_1 + R_2 + R_3 = 0$

$$\otimes = -\frac{4}{s+1} + \frac{2}{s+2} + \frac{2}{s}$$

$$\rightarrow y(t) = -4\cdot e^{-t} + 2\cdot e^{-2t} + 2\cdot\delta_{-1}(t)$$

Sappiamo che il denominatore della f.d.t è il polinomio caratteristico della matrice A

$$D(s) = (s+1)(s+2) = p(s) \to p(s)=0 \iff \begin{matrix} s=-1 \\ s=-2 \end{matrix}$$

i poli della f.d.t. sono gli autovalori del sistema entrambi sono (reali) a parte reale negativa.

$$e^{\lambda_k t} \xrightarrow[t \to +\infty]{} 0 \quad Re(\lambda_k) < 0 \Rightarrow \text{il sistema è A.S.}$$

$\Rightarrow$ esiste la risposta armonica

$$u(t) = \sin(t) \to \begin{cases} M = 1 \\ \varphi = 0 \\ \omega = 1 \end{cases}$$

$$y_{arm}(t) = |W(j)| \cdot \sin(t + \langle W(j) \rangle)$$

$$W(j) = \frac{4}{(j+1)(j+2)} = \frac{4}{-1+2+3j} = \frac{4}{1+3j} = \frac{4}{1+9}(1-j3) = \frac{4}{10}(1-j3) =$$

$$= \frac{2}{5}\sqrt{1+3^2} \cdot e^{j\, atg\left(\frac{-3}{1}\right)}$$

$$|W(j)| = \frac{2}{5}\sqrt{10} \qquad \langle W(j) \rangle = -atg(3)$$

$$\to y_{arm}(t) = \frac{2}{5}\sqrt{10} \cdot \sin(t \cancel{+} - atg(3))$$