Problema 6

$$\boxed{\begin{aligned}\dot{x}_1 &= x_1(x_2+2) + k\cdot x_1 \\ \dot{x}_2 &= -x_1^2 - x_2 - 2\end{aligned}}$$

$$\boxed{\dot{x}(t) = f(x(t))}$$

$$\boxed{\begin{aligned} x_e &= (0,-2) \\ f(x_e) &= 0\end{aligned}}$$

$$\frac{df}{dx} = J(x) = \begin{bmatrix} x_2+2+k & x_1 \\ -2x_1 & -1 \end{bmatrix} \qquad J(x_e) = \begin{bmatrix} k & 0 \\ 0 & -1 \end{bmatrix}$$

$\boxed{\lambda_1 = k, \ \lambda_2 = -1}$

$k > 0$ Instabile

$k < 0$ Asintoticamente stabile

$k = 0$ Nessuna conclusione con la linearizzazione

Un Lyapunov $V(x) = \frac{1}{2} x_1^2 + \frac{\beta}{2}\cdot(x_2+2)^2$ $\quad V(x_e) = 0$

$$\frac{dV}{dx}\cdot f(x) = \dot{V}(x) = x_1^2\cdot(x_2+2) + \beta\cdot(x_2+2)\left(-x_1^2 - (x_2+2)\right)$$

$$= x_1^2\cdot(x_2+2) - \beta x_1^2\cdot(x_2+2) - \beta\cdot(x_2+2)^2$$

let $\beta = 1$

$\dot{V}(x) = -\beta\cdot(x_2+2)^2 \leq 0$ $\quad \left[V(x_1,-2) = 0 \quad \forall x_1 \in \mathbb{R} \right]$

Semplicemente stabile $\;\; \mu \;\; k = 0$

[$\dot{V}(x)$ Semidefinita Negativa]