PROBLEMA 1

$$W_{AP}(s) = K \frac{s+10}{s(s-1)^2}\Bigg|_{K=1} = \frac{10\left(1+\frac{s}{10}\right)}{s(1-s)^2}$$

- guadagno $= 10$

  MODULO : $20 \log_{10}(10) = 20\,dB$ punto di partenza del diagramma dei moduli

  FASE : $\angle 1^0 = 0$ rad

- Termine monomio al denominatore : s

  MODULO : $-20 \frac{dB}{dec}$ in $[0, +\infty)$

  SFASAMENTO GLOBALE di $-\frac{\pi}{2}$ rad

- Termine binomio al numeratore : $\left(1+\frac{s}{10}\right) \to \omega_1 = 10$

  MODULO : $+20 \frac{dB}{dec}$ in $[10, +\infty)$

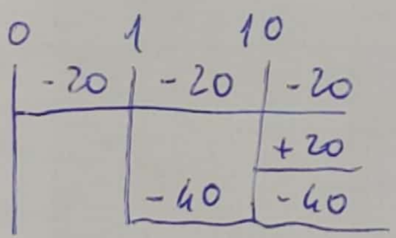
  FASE : $+\frac{\pi}{4} \frac{rad}{dec}$ in $[1, 100]$

- Termine binomio doppio al denominatore : $(1-s)^2 \to \omega_2 = 1$

  MODULO : $-40 \frac{dB}{dec}$ in $[1, +\infty)$

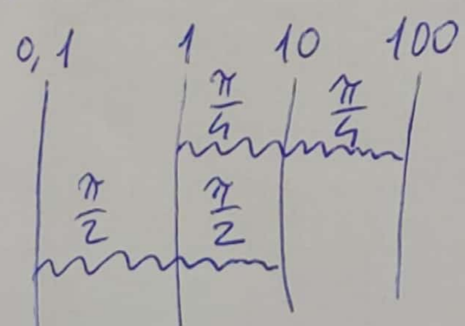
  FASE : $+\frac{\pi}{2} \frac{rad}{dec}$ in $[0{,}1 \,;\, 10]$

DIAGRAMMA DEI MODULI

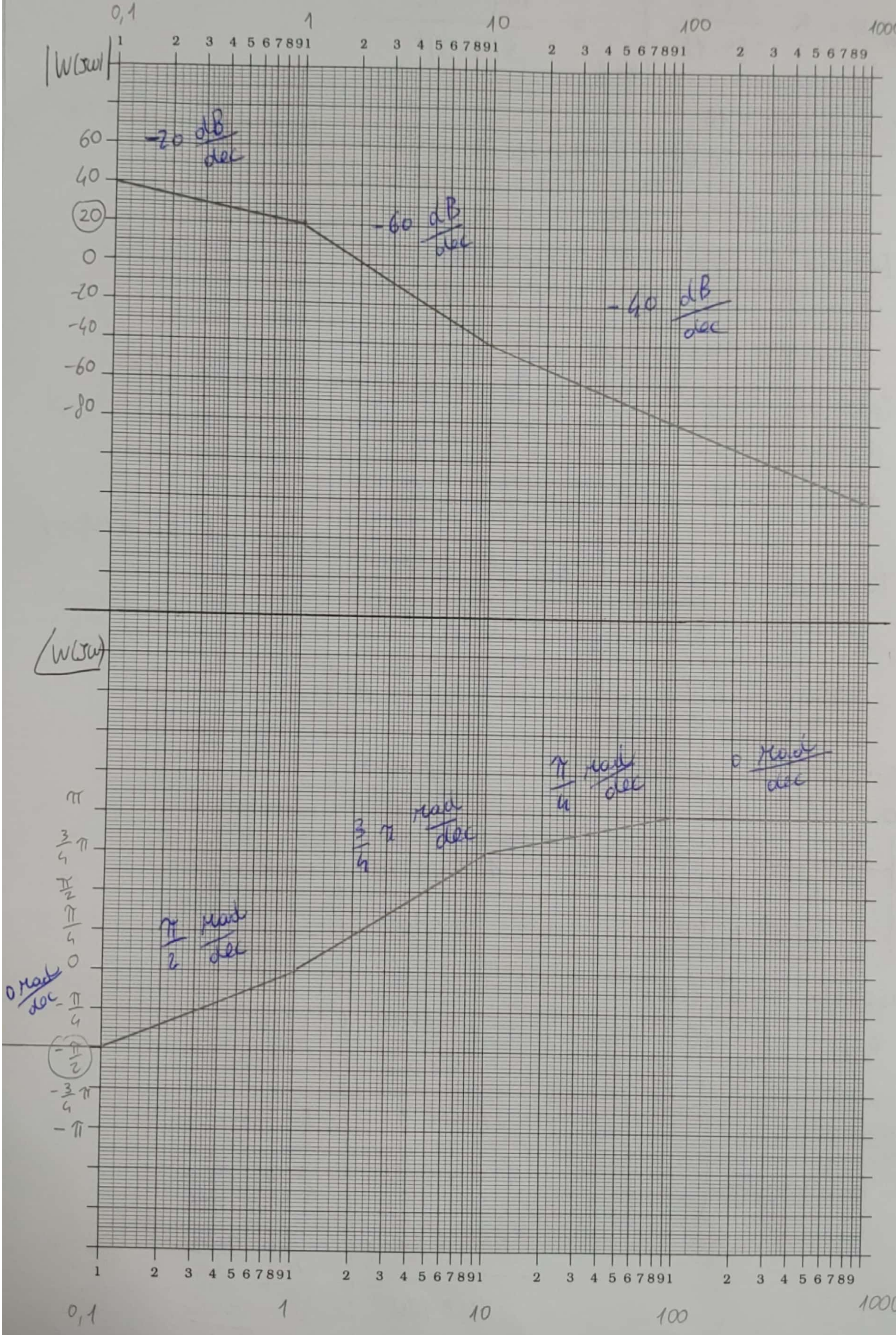


INTERVALLO	PENDENZA
$\omega < 1$	$-20 \frac{dB}{dec}$
$1 < \omega < 10$	$-60 \frac{dB}{dec}$
$\omega > 10$	$-40 \frac{dB}{dec}$

DIAGRAMMA DELLE FASI

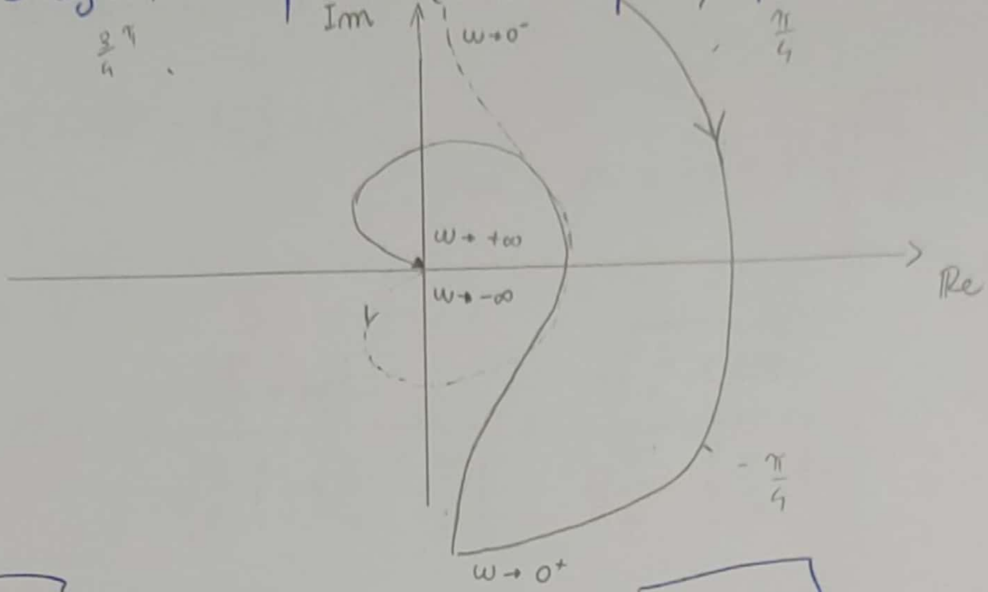


INTERVALLO	PENDENZA
$\omega < 0{,}1$	$0 \frac{rad}{dec}$
$0{,}1 < \omega < 1$	$\frac{\pi}{2} \frac{rad}{dec}$
$1 < \omega < 10$	$\frac{3}{4}\pi \frac{rad}{dec}$
$10 < \omega < 100$	$\frac{\pi}{4} \frac{rad}{dec}$
$\omega > 100$	$0 \frac{rad}{dec}$

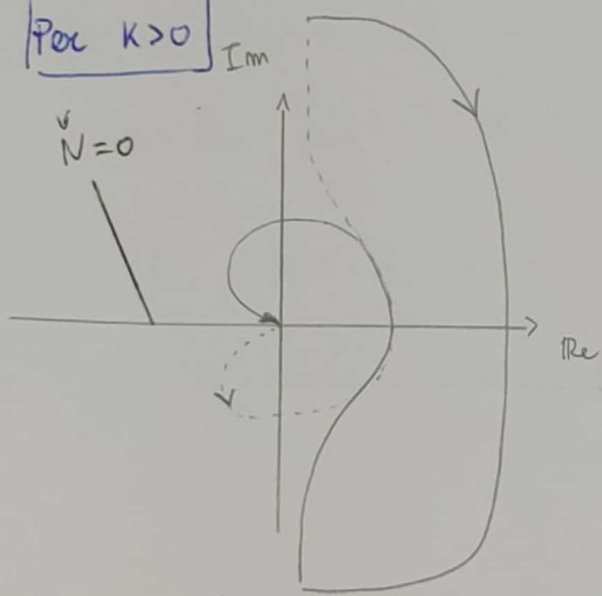


0,1
1
10
100
1000
|W(jω)|
60
40
20
0
-20
-40
-60
-80
-20 dB/dec
-60 dB/dec
-40 dB/dec
∠W(jω)
π
3/4 π
π/2
π/4
0
-π/4
-π/2
-3/4 π
-π
0 rad/dec
π/2 rad/dec
3/4 π rad/dec
π/4 rad/dec
0 rad/dec
0,1
1
10
100
1000

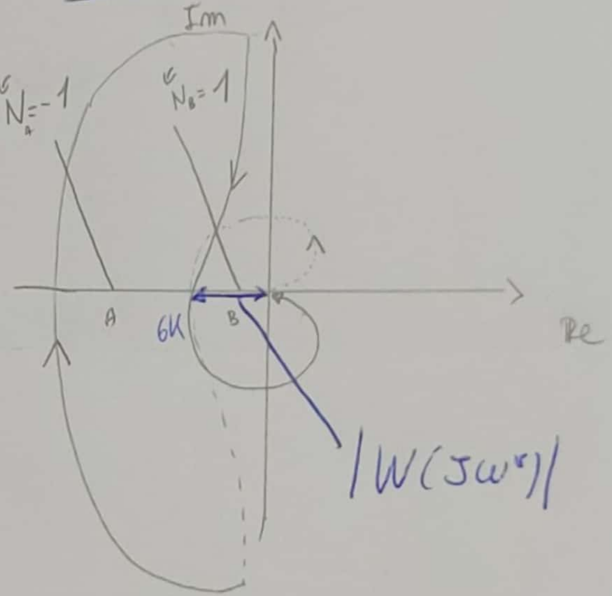
Diagramma polare (o di Nyquist) per $K=1$



$\boxed{\text{Per } K>0}$



$\boxed{\text{Per } K<0}$



$P_{CH} = P_{AP} - \check{N} = 2 - 0 = 2$ INSTABILE

ora calcolo la pulsazione di attraversamento ω^* t.c.

$$\text{Im}\{W(j\omega)\}=0$$

$$\text{Im}\left\{\frac{j\omega+10}{j\omega(j\omega-1)^2}\right\}=0 \to \text{Im}\left\{\frac{j\omega+10}{j\omega(-\omega^2+1-2j\omega)}\right\}=0$$

$$\text{Im}\left\{\frac{j\omega+10}{-j\omega^3+j\omega+2\omega^2}\right\}=0 \to \text{Im}\left\{\frac{[j\omega+10][2\omega^2-j(-\omega^3+\omega)]}{\underbrace{[2\omega^2+j(-\omega^3+\omega)][2\omega^2-j(-\omega^3+\omega)]}_{\text{reale}}}\right\}=0$$

$$\text{Im}\{[j\omega+10][2\omega^2+j\omega^3-j\omega]\}=0$$

$$\text{Im}\{2j\omega^3-\cancel{\omega^4}+\cancel{\omega^2}+20\omega^2+10j\omega^3-10j\omega\}=0$$

$$Im\{12j\omega^3 - 10j\omega\} = 0$$

$$\omega(12\omega^2 - 10) = 0 \quad \begin{cases} \omega = 0 \\ 12\omega^2 = 10 \rightarrow \omega = \pm\sqrt{\frac{5}{6}} \end{cases}$$

$$\omega^* = \sqrt{\frac{5}{6}}$$

$$|W(j\omega^*)| = \frac{\left|j\sqrt{\frac{5}{6}} + 10\right|}{\left|j\sqrt{\frac{5}{6}}\left(j\sqrt{\frac{5}{6}} - 1\right)^2\right|} = \frac{\sqrt{\frac{5}{6} + 100}}{\sqrt{\frac{5}{6}} \cdot \left(\frac{5}{6} + 1\right)} = 6$$

CASO A: $-1 < 6K \rightarrow 6K > -1 \rightarrow K > -\frac{1}{6}$

$P_{CH} = P_{AP} - \overset{\circ}{N}_A = 2 - (-1) = 3$ INSTABILE

CASO B: $6K < -1 \rightarrow K < -\frac{1}{6}$

$P_{CH} = P_{AP} - \overset{\circ}{N}_B = 2 - 1 = 1$ INSTABILE

Denominatore della f.d.t. a ciclo chiuso

$$W_{CH}(s) = \frac{W(s)}{1+W(s)} = \frac{\frac{N(s)}{D(s)}}{1+\frac{N(s)}{D(s)}} = \frac{\frac{N(s)}{D(s)}}{\frac{D(s)+N(s)}{D(s)}} = \frac{N(s)}{D(s)+N(s)}$$

$$D_{CH}(s) = D(s)+N(s) = s(s-1)^2 + ks + 10k =$$

$$= s(s^2+1-2s) + ks + 10k =$$

$$= s^3 + s - 2s^2 + ks + 10k =$$

$$= s^3 - 2s^2 + (1+k)s + 10k$$

Costruisco la tabella di Routh

$$\begin{array}{c|cc} 3 & 1 & 1+k \\ 2 & -2 & 10k \\ 1 & 6k+1 & \\ 0 & 10k & \end{array}$$

$$\frac{1}{2}\begin{vmatrix} 1 & 1+k \\ -2 & 10k \end{vmatrix} = \frac{1}{2}\left(10k+2+2k\right) =$$

$$= \frac{1}{2}\left(12k+2\right) = 6k+1$$

$$\begin{array}{c|ll} 3 & 1 & + \\ 2 & -2 & - \\ 1 & 6k+1 & \text{studio il segno } 6k > -1 \rightarrow k > -\frac{1}{6} \\ 0 & 10k & \quad // \qquad // \qquad k > 0 \end{array}$$

		$-\frac{1}{6}$	0
1	+	+	+
-2	−	−	−
$6k+1$	−	+	+
$10k$	−	−	+
	1V	3V	2V

Per $k > 0$ $\quad P_{CH} = 2 \rightarrow$ SISTEMA A CICLO CHIUSO INSTABILE

Per $k \in [-\frac{1}{6}, 0]$ $\quad P_{CH} = 3 \rightarrow$ // // //

Per $k < -\frac{1}{6}$ $\quad P_{CH} = 1 \rightarrow$ // // //

PROBLEMA 2 TEMPO CONTINUO

$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C = [1 \quad 0]$

$\lambda_1 = -4 + i$ $\quad r_1 = \begin{bmatrix} 2i \\ 1 \end{bmatrix}$ $\quad r_2 = \begin{bmatrix} -2i \\ 1 \end{bmatrix}$ essendo gli autovalori complessi coniugati.

$\lambda_2 = -4 - i$

$Re(\lambda_1), Re(\lambda_2) < 0$ MODI ASINTOTICAMENTE STABILI

$$R = \begin{bmatrix} 2i & -2i \\ 1 & 1 \end{bmatrix}$$

Autovettori sinistri

$$L = R^{-1} = \frac{1}{det(R)} \begin{bmatrix} 1 & 2i \\ -1 & 2i \end{bmatrix} = , \quad det(R) = 2i + 2i = 4i$$

$$= \frac{1}{4i} \begin{bmatrix} 1 & 2i \\ -1 & 2i \end{bmatrix} = \begin{bmatrix} \frac{1}{4i} & \frac{1}{2} \\ -\frac{1}{4i} & \frac{1}{2} \end{bmatrix} \begin{matrix} - l_1^t \\ - l_2^t \end{matrix}$$

Verificare che $l_1^T \cdot r_1 = 1$
$l_1^T \cdot r_2 = 0$

ECCITABILITÀ

$l_1^T B = \left[\frac{1}{4i} \quad \frac{1}{2} \right] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{4i} \neq 0$ ECCITABILE (idem per $l_2^t \cdot B$)

OSSERVABILITÀ

$C \cdot r_1 = [1 \quad 0] \begin{bmatrix} 2i \\ 1 \end{bmatrix} = 2i \neq 0$ OSSERVABILE (idem per $C \cdot r_2$)

$$A = R \Lambda L = \begin{bmatrix} 2i & -2i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -4+i & 0 \\ 0 & -4-i \end{bmatrix} \begin{bmatrix} \frac{1}{4i} & \frac{1}{2} \\ -\frac{1}{4i} & \frac{1}{2} \end{bmatrix} =$$

$$= \begin{bmatrix} -8i-2 & 8i-2 \\ -4+i & -4-i \end{bmatrix} \begin{bmatrix} \frac{1}{4i} & \frac{1}{2} \\ -\frac{1}{4i} & \frac{1}{2} \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{8\cancel{i}}{4\cancel{i}} - \cancel{\frac{2}{4i}} - \frac{8i}{4i} + \cancel{\frac{2}{4i}} & -\frac{8\cancel{i}}{2} - 1 + \frac{8\cancel{i}}{2} - 1 \\ -\cancel{\frac{4}{4i}} + \frac{i}{4i} + \cancel{\frac{4}{4i}} + \frac{i}{4i} & -\frac{4}{2} + \cancel{\frac{i}{2}} - \frac{4}{2} - \cancel{\frac{i}{2}} \end{bmatrix} =$$

$$= \begin{bmatrix} -4 & -2 \\ \frac{1}{2} & -4 \end{bmatrix} = A$$

Calcolo più veloce

$$A = 2\,Re\left((-4+i)\cdot\begin{bmatrix} 2i \\ 1 \end{bmatrix}\cdot\begin{bmatrix} \frac{1}{4i} & \frac{1}{2} \end{bmatrix}\right)$$

$$\Phi(t) = e^{At} = \cancel{2}\,Re\left\{ e^{(-4+i)t} \begin{bmatrix} 2i \\ 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\cancel{4}_{2}i} & \frac{1}{\cancel{2}} \end{bmatrix} \right\} =$$

$$= Re\left\{ e^{(-4+i)t} \begin{bmatrix} 2i \\ 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2i} & 1 \end{bmatrix} \right\} =$$

$$= Re\left\{ e^{-4t} e^{it} \begin{bmatrix} 1 & 2i \\ \frac{1}{2i} & 1 \end{bmatrix} \right\} =$$

$$e^{it} = \cos(t) + i \sin(t)$$

$$= e^{-4t}\,Re\left\{ (\cos(t) + i\sin(t)) \begin{bmatrix} 1 & 2i \\ \frac{1}{2i} & 1 \end{bmatrix} \right\} =$$

$$= e^{-4t} \begin{bmatrix} \cos(t) & -2\sin(t) \\ \frac{1}{2}\sin(t) & \cos(t) \end{bmatrix} = \begin{bmatrix} e^{-4t}\cos(t) & -2e^{-4t}\sin(t) \\ \frac{1}{2}e^{-4t}\sin(t) & e^{-4t}\cos(t) \end{bmatrix} =$$

$$= \begin{bmatrix} *11 & *12 \\ *21 & *22 \end{bmatrix}$$

Calcolo di controllo

$$e^{At}\Big|_{t=0} = I$$

$$W(t) = C\, e^{At} B = [1 \quad 0] \begin{bmatrix} *11 & *12 \\ *21 & *22 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} =$$

$$= \left[e^{-4t}\cos(t) \quad -2\, e^{-4t}\sin(t) \right] \begin{bmatrix} 1 \\ 0 \end{bmatrix} =$$

$$= e^{-4t}\cos(t)$$

$$W(s) = \mathcal{L}\{w(t)\} = \frac{s+4}{(s+4)^2+1} = \frac{s+4}{s^2+16+8s+1} =$$

$$= \frac{s+4}{s^2+8s+17} \; =$$

$$s_{1,2} = \frac{-8 \pm \sqrt{-4}}{2} = \frac{-8 \pm 2i}{2} = \begin{cases} -4-i \\ -4+i \end{cases}$$

E' solo un calcolo di controllo, in quanto
i due poli calcolati devono essere gli autovalori della A

$$= \frac{s+4}{(s+4-i)(s+4+i)} \qquad U(s) = \frac{1}{s}$$

$$Y_f(s) = W(s)\,U(s) = \frac{s+4}{(s+4-i)(s+4+i)} \cdot \frac{1}{s} = \frac{R_1}{s+4-i} + \frac{R_1^*}{s+4-i} + \frac{R_2}{s}$$

$$R_1 = \lim_{s \to -4+i} \frac{s+4}{s(s+4+i)} = \frac{-4+i+4}{(-4+i)(-4+i+4+i)} =$$

$$= \frac{i}{(-4+i)2i} = \frac{1}{-8+2i} = \frac{-8-2i}{(-8+2i)(-8-2i)} =$$

$$= \frac{-8-2i}{64+\cancel{16i}-\cancel{16i}+4} = -\frac{8}{68} - \frac{2i}{68} = -\frac{2}{17} - \frac{1}{34} i$$

$$R_1^* = -\frac{2}{17} + \frac{1}{34} i$$

$$R_2 = \lim_{s \to 0} \frac{s+4}{s^2+8s+17} = \frac{4}{17}$$

$$Y_f(s) = \frac{4}{17}\,\frac{1}{s} + \left(-\frac{2}{17} - \frac{1}{34}\,i\right)\frac{1}{s+4-i} + \left(-\frac{2}{17} + \frac{1}{34}\,i\right)\frac{1}{s+4+i}$$

$s+(4-i)$ $\qquad$ $s+(4+i)$

$$y_f(t) = \mathcal{L}^{-1}\left\{Y_f(s)\right\} = \frac{4}{17}\,\delta_{-1}(t) + \left(-\frac{2}{17} - \frac{1}{34}\,i\right)e^{-(4-i)t} + \left(-\frac{2}{17} + \frac{1}{34}\,i\right)e^{-(4+i)t} =$$

$$= \frac{4}{17}\,\delta_{-1}(t) \cdot -\frac{2}{17}\,e^{-4t}\left(e^{it} + e^{-it}\right)\frac{2}{2} - e^{-4t}\,\frac{1}{17}\,i\left(\frac{e^{it} - e^{-it}}{2}\right)\frac{i}{i} =$$

$$= \frac{4}{17}\,\delta_{-1}(t) - \frac{4}{17}\,e^{-4t}\cos(t) + e^{-4t}\,\frac{1}{17}\sin(t)$$

Poiché il legame diretto è assente $(D=0)$ la risposta forzata deve essere zero per $t=0$

$$y_f(0) = C\,\underset{\overset{\|}{0}}{x(0)} = 0$$

Proprietà verificata calcolando $y_f(0)$

$$y(t) = \frac{1}{17}\left(4 - 4e^{-4t}\cos(t) + \sin(t)\right)\cdot\delta_{-1}(t)$$

PROBLEMA 3 TEMPO DISCRETO

$$W(z) = \frac{z}{z - \frac{1}{2}}$$

$u(t) = \sqrt{5} \sin\left(\frac{\pi}{2} t\right)$

$$W(t) = \mathcal{Z}^{-1}\{W(z)\} = \left(\frac{1}{2}\right)^t$$

$M = \sqrt{5}$, $\varphi = 0$, $\omega = \frac{\pi}{2}$

$$W(0) = 1$$

$$Y_{arm}(t) = M \left| W(e^{j\omega}) \right| \sin\left(\omega t + \varphi + \angle W(e^{j\omega})\right)$$

$\cancel{|W(j\omega)|}$

$$|W(e^{j\omega})| = \frac{|e^{j\omega}|}{\left|e^{j\omega} - \frac{1}{2}\right|} =$$

$$e^{j\omega} \overset{\omega = \frac{\pi}{2}}{=} \cos\left(\frac{\pi}{2}\right) + j \sin\left(\frac{\pi}{2}\right) = j$$

$$= \frac{|j|}{\left|j - \frac{1}{2}\right|} = \frac{1}{\sqrt{1 + \frac{1}{4}}} = \frac{2\sqrt{5}}{5}$$

$$\angle \frac{j}{j - \frac{1}{2}} = \angle j - \angle j - \tfrac{1}{2} = \frac{\pi}{2} - \operatorname{arctg}(-2) - \pi \simeq -0{,}46 \text{ rad}$$

$$Y_{arm}(t) = \sqrt{5} \cdot \frac{2\sqrt{5}}{5} \sin\left(\frac{\pi}{2} t - 0{,}46\right) =$$

$$= 2 \sin\left(\frac{\pi}{2} t - 0{,}46\right)$$