Ex. 1

$$W(s) = k \frac{50(s+1)}{(s+5)(s^2+100)}$$

$p_1 = -5 \Rightarrow Re(p_1) < 0$

$p_{2,3} = \pm j10 \rightarrow Re(p_{2,3}) = 0$

Per $k = 1$ ho:

$$\tilde{W}(s) = \frac{50(s+1)}{(s+5)(s^2+100)} = \frac{\cancel{50}\left(1+\frac{s}{1}\right)}{\underset{10}{\cancel{500}}\left(1+\frac{s}{5}\right)\left(1+\frac{2\xi s}{10}+\frac{s^2}{100}\right)} =$$

$$= \frac{1}{10} \frac{\left(1+\frac{s}{1}\right)}{\left(1+\frac{s}{5}\right)\left(1+\frac{2\xi s}{10}+\frac{s^2}{100}\right)}, \quad \xi = 0$$

$W(0) = 1/10 \qquad |W(0)|_{dB} = 20\log_{10} 1 - 20\log_{10} 10 = 0 - 20\,dB = -20\,dB$

$\langle W(0) \rangle = 0$ RAD

Per $\omega \Rightarrow +\infty$, $W(j\omega) \approx \frac{\cancel{j\omega}}{\cancel{j\omega}(-\omega^2)} = -\frac{1}{\omega^2}$, $\langle -\frac{1}{\omega^2} \rangle = \pm\pi$

TERMINE BINOMIO NUMERATORE: $\omega_t = 1$ RAD/s

MODULI		FASI	
$\omega \in (0, 1)$	0 dB/dec	$\omega \in (0, 0.1)$	0 RAD/dec
$\omega \in (1, +\infty)$	+20 dB/dec	$\omega \in (0.1, 10)$	$+\pi/4$ RAD/dec
		$\omega \in (10, +\infty)$	0 RAD/dec

TERMINE BINOMIO DENOMINATORE: $\omega_t = 5$ RAD/s

MODULI		FASI	
$\omega \in (0, 5)$	0 dB/dec	$\omega \in (0, 0.5)$	0 ~~dB~~/dec
$\omega \in (5, +\infty)$	-20 dB/dec	$\omega \in (0.5, 50)$	$-\pi/4$ RAD/dec
		$\omega \in (50, +\infty)$	0 ~~dB~~/dec

TERMINE TRINOMIO DENOMINATORE: $\omega_n = 10$ RAD/S

MODULI

$\omega \in (0, 10)$ 0 dB/dec

$\omega \in (10, +\infty)$ -40 dB/dec

FASI

SFASAMENTO ISTANTANEO DI $-\pi$ in $\omega_n = 10$ RAD/s

RIASSUMENDO

MODULI

$|W(0)|_{dB} = -20\,dB$

$\omega \in (0, 1)$ 0 dB/dec

$\omega \in (1, 5)$ +20 dB/dec

$\omega \in (5, 10)$ 0 dB/dec

$\omega \in (10, +\infty)$ -40 dB/dec

FASI

SFASAMENTO ISTANTANEO DI $-\pi$ in $\omega_n = 10$

$\angle W(0) = 0$

$\omega \in (0, 0.1)$ 0 RAD/dec

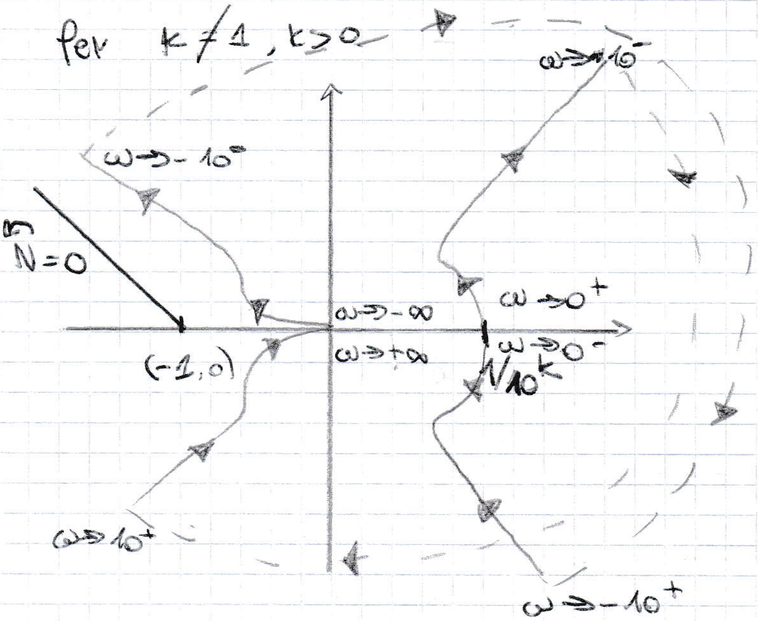
$\omega \in (0.1, 0.5)$ $+\pi/4$ RAD/dec

$\omega \in (0.5, 10)$ 0 RAD/dec

$\omega \in (10, 50)$ $-\pi/4$ RAD/dec

$\omega \in (50, +\infty)$ 0 RAD/dec

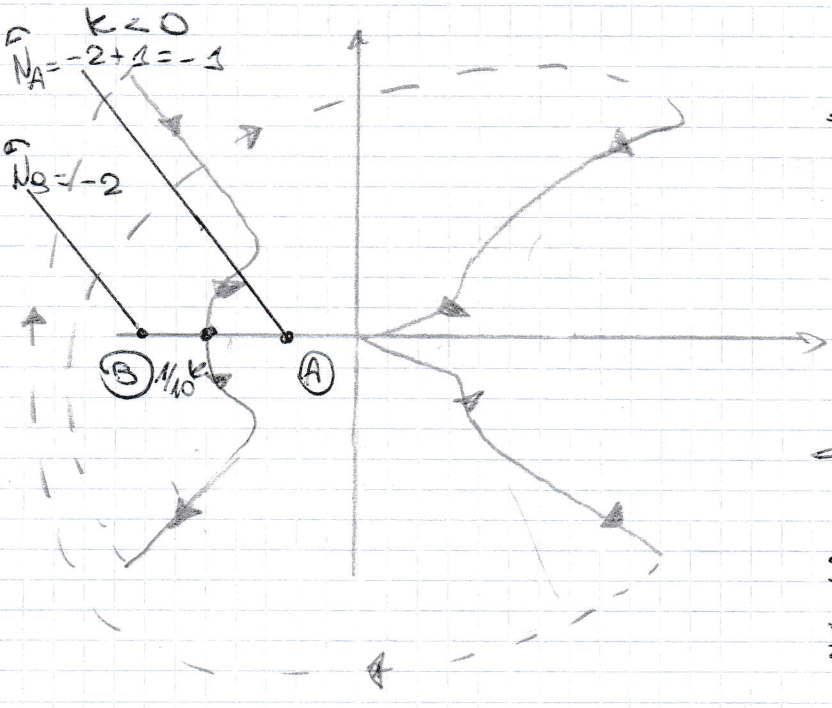
per $k \neq 1$, $k > 0$



$P_{CH} = 0 - 0 = 0$

$\Rightarrow W_{CH}$ ha zero poli a parte reale > 0 ed è A.S. $\forall k \in (0, +\infty)$

$k < 0$



(A) $P_{CH} = 0 - (-1) = 1$

$\Rightarrow W_{CH}$ ha 1 polo a parte reale > 0 ed è INST.

Se $\frac{1}{10}k < -1 \Leftrightarrow k < -10$

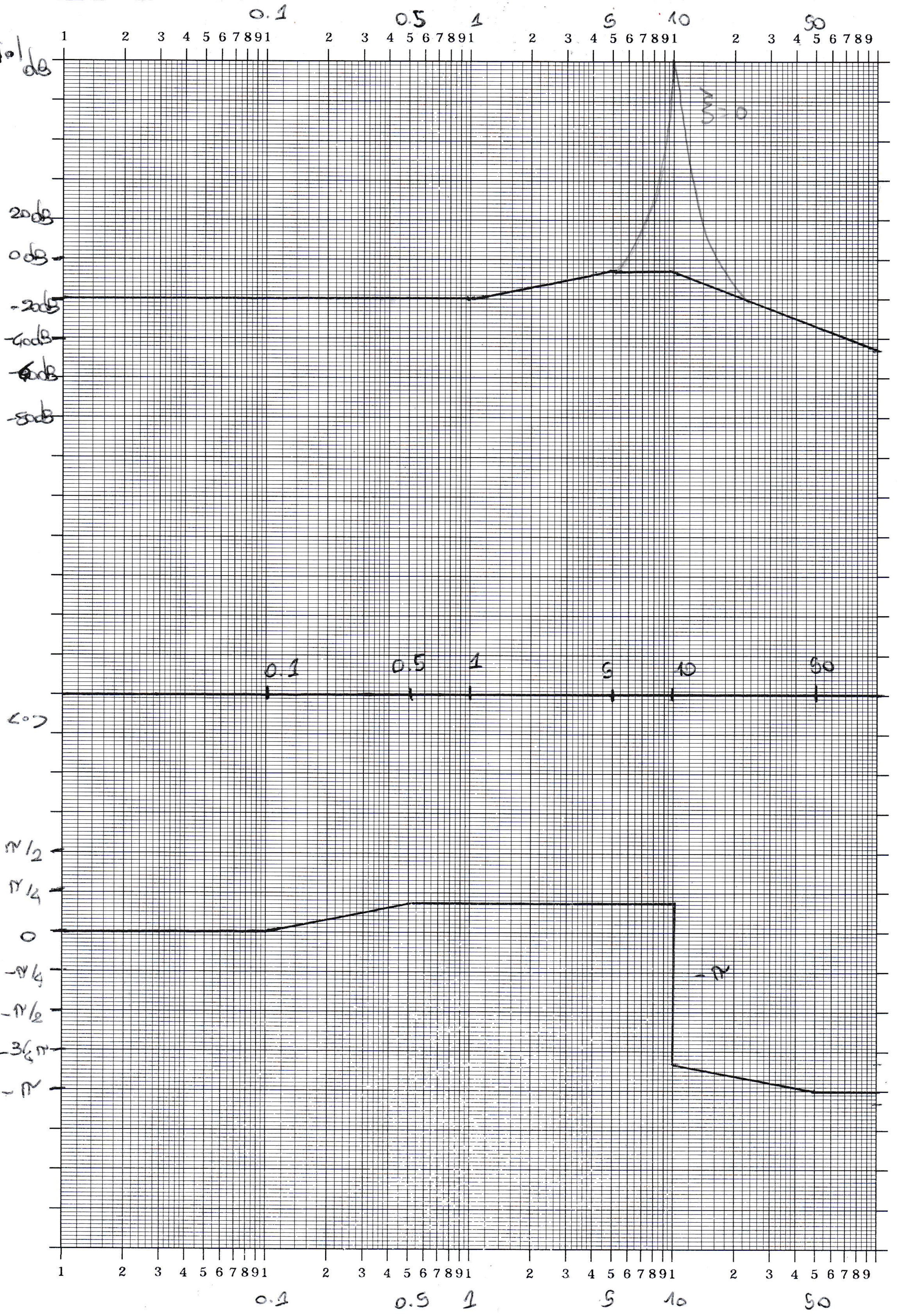
$\forall k \in (-\infty, -10)$

(B) $P_{CH} = 0 - (-2) = 2$

$\Rightarrow W_{CH}$ ha 2 poli a parte reale > 0 ed è INST.

Se $\frac{1}{10}k > -1 \Leftrightarrow k > -10$

$\forall k \in (-10, 0)$



0.1
0.5
1
5
10
50
dB
20dB
0dB
-20dB
-40dB
-60dB
-80dB
ξ=0
<φ
π/2
π/4
0
-π/4
-π/2
-3/4π
-π
-π

Con Routh

$$D_{CH} = D_{AP} + k N_{AP} = s^3 + 100s + 5s^2 + 500 + 50ks + 50k =$$
$$= s^3 + 5s^2 + 50(2+k)s + 50k + 500$$

3	1	$50(2+k)$	0
2	5	$50k+500$	0
1	$40k$		
0	$50k+500$		

$$\frac{\begin{vmatrix} 1 & 50(2+k) \\ 5 & 50k+500 \end{vmatrix}}{-5} = -\frac{1}{5}\left[50k + \cancel{500} - \cancel{500} - 250k\right] = -\frac{1}{5}(-200k) = 40k$$

	-10	0	
1	+	+	+
5	+	+	+
$40k$	−	−	+
$50k+500$	−	+	+

$40k > 0 \iff k > 0$

$50k + 500 = 50(k+10) > 0 \iff k > -10$

1 c.s. 2 c.s. 0 c.s. $\rightarrow$ 0 poli a $Re > 0$ $\forall k \in (0, +\infty)$ W_{CH} A.S.

1 polo a $Re > 0$ $\forall k \in (-\infty, -10)$ W_{CH} INST

2 poli a $Re > 0$ $\forall k \in (-10, 0)$ W_{CH} INST

Nyquist confermato

Ex. 2

T.D.

$$A = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda-2 & 2 \\ -2 & \lambda-2 \end{vmatrix} = \lambda^2 - 4\lambda + 4 + 4 = \lambda^2 - 4\lambda + 8 = 0$$

$$\Delta = 16 - 32 = -16$$

$$\lambda_{1,2} = \frac{4 \pm j4}{2} \quad \lambda_1 = 2 + j2 \quad \lambda_2 = 2 - j2 \quad \lambda_2 = \lambda_1^*$$

$|\lambda_1| = \sqrt{8} > 1$

$|\lambda_2| = \sqrt{8} > 1$ $\Rightarrow$ Entrambi i modi naturali associati ai 2 autovalori sono instabili

$r_1 : [\lambda_1 I - A]\, r_1 = \begin{bmatrix} j2 & 2 \\ -2 & j2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow r_1 = \begin{bmatrix} j \\ 1 \end{bmatrix}$

$r_2 = r_1^* = \begin{bmatrix} -j \\ 1 \end{bmatrix}$

$R = \begin{bmatrix} j & -j \\ 1 & 1 \end{bmatrix}, \; L = R^{-1} \Rightarrow L = \frac{1}{2j} \begin{bmatrix} 1 & -1 \\ j & j \end{bmatrix}^T = \frac{1}{2j} \begin{bmatrix} 1 & j \\ -1 & j \end{bmatrix} =$

$= \begin{bmatrix} 1/2j & 1/2 \\ -1/2j & 1/2 \end{bmatrix} = \begin{bmatrix} -j/2 & 1/2 \\ j/2 & 1/2 \end{bmatrix} \begin{matrix} \to l_1^T \\ \to l_2^T \end{matrix}$

$C r_1 = [1 \;\; 1] \begin{bmatrix} j \\ 1 \end{bmatrix} = 1 + j \neq 0$ $\Rightarrow$ Entrambi i modi naturali sono osservabili in uscita

$C r_2 = [1 \;\; 1] \begin{bmatrix} -j \\ 1 \end{bmatrix} = 1 - j \neq 0$

$l_1^T B = [-j/2 \;\; 1/2] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = -j/2 \neq 0$ $\Rightarrow$ Entrambi i modi naturali sono eccitabili per impulsi in ingresso

$l_2^T B = [j/2 \;\; 1/2] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = j/2 \neq 0$

$\Phi(t) = A^t = \lambda_1^t r_1 l_1^T + \lambda_1^{*t} r_1^* l_1^{T*} =$

$= 2\,\mathrm{Re}\left\{ \lambda_1^t r_1 l_1^T \right\} = 2\,\mathrm{Re}\left\{ (2+j2)^t \begin{bmatrix} j \\ 1 \end{bmatrix} [-j/2 \;\; 1/2] \right\} =$

$= 2\,\mathrm{Re}\left\{ (2)^t (1+j)^t \begin{bmatrix} 1/2 & j/2 \\ -j/2 & 1/2 \end{bmatrix} \right\} = 2\,\mathrm{Re}\left\{ 2^t \cdot (\sqrt{2})^t e^{j\pi/4\, t} \begin{bmatrix} 1/2 & j/2 \\ -j/2 & 1/2 \end{bmatrix} \right\} =$

$1 + j = \sqrt{2}\, e^{j\pi/4}$

$= 2\,\mathrm{Re}\left\{ \begin{bmatrix} \frac{(\sqrt{8})^t}{2}\cos(\pi/4\, t) + j\frac{(\sqrt{8})^t}{2}\sin(\pi/4\, t) & \frac{j(\sqrt{8})^t}{2}\cos(\pi/4\, t) - \frac{(\sqrt{8})^t}{2}\sin(\pi/4\, t) \\ -\frac{j(\sqrt{8})^t}{2}\cos(\pi/4\, t) + \frac{(\sqrt{8})^t}{2}\sin(\pi/4\, t) & \frac{(\sqrt{8})^t}{2}\cos(\pi/4\, t) + j\frac{(\sqrt{8})^t}{2}\sin(\pi/4\, t) \end{bmatrix} \right\}$

$2^t (\sqrt{2})^t = (\sqrt{8})^t$

$= \begin{bmatrix} (\sqrt{8})^t \cos(\pi/4\, t) & -(\sqrt{8})^t \sin(\pi/4\, t) \\ (\sqrt{8})^t \sin(\pi/4\, t) & (\sqrt{8})^t \cos(\pi/4\, t) \end{bmatrix} \to \Phi(0) = I$

$$w(t) = CA^{t-1}B + D \,, \quad D = 0$$

$$w(1) = CA^{0}B = CIB = CB = [1 \quad 1]\begin{bmatrix}1\\0\end{bmatrix} = 1$$

$$w(2) = CA^{2-1}B = CAB = [1 \quad 1]\begin{bmatrix}2 & -2\\2 & 2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = 4$$

$$w(2) = \cancel{(\sqrt{8})^{t}\left[\cos\left(\frac{\pi}{4}t\right) + \sin\left(\frac{\pi}{4}t\right)\right]} \; (\sqrt{8})\cos\left(\frac{\pi}{4}\right) + (\sqrt{8})\sin\left(\frac{\pi}{4}\right) =$$

$$= \cancel{2}\sqrt{2}\,\frac{\sqrt{2}}{\cancel{2}} + \cancel{2}\sqrt{2}\,\frac{\sqrt{2}}{\cancel{2}} = 4$$

$$w(t) = CA^{t-1}B = [1 \quad 1]\left[\; A^{t-1} \;\right]\begin{bmatrix}1\\0\end{bmatrix} =$$

$$= (\sqrt{8})^{t-1}\cdot\left[\cos\left(\frac{\pi}{4}(t-1)\right) + \sin\left(\frac{\pi}{4}(t-1)\right)\right]$$

$$W(z) = \frac{1}{z}\, Z\left\{(\sqrt{8})^{t}\cos\left(\frac{\pi}{4}t\right) + (\sqrt{8})^{t}\sin\left(\frac{\pi}{4}t\right)\right\}$$

$$= \frac{1}{z}\, Z\left\{(\sqrt{8})^{t}\cos\left(\frac{\pi}{4}t\right)\right\} + \frac{1}{z}\, Z\left\{(\sqrt{8})^{t}\sin\left(\frac{\pi}{4}t\right)\right\}$$

$$= \frac{1}{z}\left.\frac{z^2 - z\cos\left(\frac{\pi}{4}\right)}{z^2 - 2z\cos\left(\frac{\pi}{4}\right) + 1}\right|_{\frac{z}{\sqrt{8}}} + \frac{1}{z}\left.\frac{z\sin\left(\frac{\pi}{4}\right)}{z^2 - 2z\cos\left(\frac{\pi}{4}\right) + 1}\right|_{\frac{z}{\sqrt{8}}}$$

$$= \left.\frac{z - \cos\left(\frac{\pi}{4}\right)}{z^2 - 2z\cos\left(\frac{\pi}{4}\right) + 1}\right|_{\frac{z}{\sqrt{8}}} + \left.\frac{\sin\left(\frac{\pi}{4}\right)}{z^2 - 2z\cos\left(\frac{\pi}{4}\right) + 1}\right|_{\frac{z}{\sqrt{8}}}$$

$$= \frac{\frac{z}{\sqrt{8}} - \frac{\sqrt{2}}{2}}{\frac{z^2}{8} - \frac{\sqrt{2}}{\sqrt{8}}z + 1} + \frac{\frac{\sqrt{2}}{2}}{\frac{z^2}{8} - \frac{\sqrt{2}}{\sqrt{8}}z + 1} = \frac{\frac{z}{\sqrt{8}} - \frac{\sqrt{2}}{2}}{\frac{z^2}{8} - \frac{z}{2} + 1} + \frac{\frac{\sqrt{2}}{2}}{\frac{z^2}{8} - \frac{z}{2} + 1} =$$

$$= \frac{\frac{2z - 4}{4\sqrt{2}}}{\frac{z^2 - 4z + 8}{8}} + \frac{\frac{\sqrt{2}}{2}}{\frac{z^2 - 4z + 8}{8}} = \frac{2z - 4}{\cancel{4}\sqrt{2}}\,\frac{\overset{2}{\cancel{8}}}{z^2 - 4z + 8} + \frac{\sqrt{2}}{\cancel{2}}\,\frac{\overset{4}{\cancel{8}}}{z^2 - 4z + 8}$$

$$= \frac{4z - 8}{\sqrt{2}(z^2 - 4z + 8)} + \frac{4\sqrt{2}}{(z^2 - 4z + 8)} = \frac{4\sqrt{2}z - 8\sqrt{2} + 4\sqrt{2}}{z^2 - 4z + 8} = \frac{4\sqrt{2}z - 4\sqrt{2}}{z^2 - 4z + 8} =$$

$$= \frac{4\sqrt{2}(z - 1)}{(z^2 - 4z + 8)}$$

Ex. 3

T.C.

$$w(t) = e^{-t} + e^{-4t}$$

$$W(s) = \frac{1}{s+1} + \frac{1}{s+4} = \frac{2s+5}{(s+1)(s+4)}$$

$$u(t) = e^{2t} \Rightarrow U(s) = \frac{1}{s-2}$$

$$Y(s) = W(s)U(s) = \frac{2s+5}{(s+1)(s+4)(s-2)} = \frac{R_1}{s+1} + \frac{R_2}{s+4} + \frac{R_3}{s-2}$$

$$R_1 = \left.\frac{2s+5}{(s+4)(s-2)}\right|_{-1} = \frac{\cancel{3}}{-\cancel{9}\,3} = -1/3$$

$$R_2 = \left.\frac{2s+5}{(s+1)(s-2)}\right|_{-4} = \frac{-3}{18} = -\frac{3}{18}$$

$$R_3 = \left.\frac{2s+5}{(s+1)(s+4)}\right|_{2} = \frac{\cancel{9}}{1\cancel{8}\,2} = \frac{1}{2}$$

$$\Rightarrow Y(s) = -\frac{1}{3}\frac{1}{s+1} - \frac{\cancel{3}\,1}{\cancel{18}\,6}\frac{1}{s+4} + \frac{1}{2}\frac{1}{s-2}$$

$$y(t) = -\frac{1}{3}e^{-t} - \frac{3}{18}e^{-4t} + \frac{1}{2}e^{2t}$$

$$u(t) = \sin(2t+\pi)$$

$\nearrow A = 1$

$\rightarrow \omega = 2$ RAD/s

$\searrow \varphi = \pi$

La risposta armonica esiste perche il ~~sistema~~ $w(t)$ ha solo poli a parte reale < 0

~~W(j2) H(j\omega)~~

$$W(j2) = \frac{4j+5}{(1+2j)(4+2j)} = \frac{4j+5}{\cancel{4}+2j+8j-\cancel{4}} = \frac{4j+5}{10j} = -j\,\frac{4j+5}{10} =$$

$$= \frac{4-5j}{10} = \frac{2}{5} - \frac{1}{2}j$$

$$|W(j2)| = \sqrt{\frac{4}{25}+\frac{1}{4}} = \sqrt{\frac{16+25}{100}} = \frac{\sqrt{41}}{10}$$

$$\angle W(j2) > = \mathrm{ARCtg}\left(\frac{-1/2}{2/5}\right) = -\mathrm{ARCtg}\left(\frac{5}{4}\right) \approx 51.34^{\circ}$$

$$y_{ARM}(t) = A|W(j\omega)| \sin(\omega t + \varphi + \angle W(j\omega))$$

$$= \frac{\sqrt{41}}{10} \sin(2t + \pi + \underbrace{51.34})$$

Ex. 4

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad C = [1 \quad 0 \quad 0 \quad 0]$$

$$\mathcal{X} \subseteq \mathbb{R}^4$$

$$R = [B | AB | A^2B | A^3B] = \begin{bmatrix} 0 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$ → ~~com~~ uguale alla 2^a RIGA

ma $\begin{vmatrix} 0 & 2 & 2 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 2 - 4 = -2 \neq 0 \Rightarrow \rho(R) = 3$

$$\dim(\mathcal{R}) = \dim(\mathrm{Im}(R)) = \rho(R) = 3$$

$$\mathcal{R} = \mathrm{Im}(R) = \mathrm{span}\left\{\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \\ 1 \end{pmatrix}\right\} = \mathrm{span}\left\{\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}\right\}$$

$$= \mathrm{span}\left\{\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}\right\} = \mathrm{span}\left\{\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}\right\}$$ → Ne costituisce anche una base

$$Q = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \quad \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \neq 0 \Rightarrow \rho(Q) = 2$$

→ uguali alla 2^a RIGA

$$\dim(\mathcal{Q}) = \dim(\ker(Q)) = n - \dim(\mathrm{Im}(Q)) = 4 - 2 = 2$$

$$x: \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow x_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad x_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\Rightarrow \mathcal{Q} = \ker(Q) = \mathrm{span}\left\{\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}\right\}$$

$$\mathcal{X}_1 = \mathcal{R} \cap \mathcal{Q}$$

un vettore $\in \mathcal{R} \cap \mathcal{Q}$ se $\in \mathcal{R}$ e $\in \mathcal{Q}$

$$v \in \mathcal{R} \Leftrightarrow v = \alpha_1 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\alpha_2 \\ \alpha_1 \\ \alpha_3 \\ \alpha_1 \end{pmatrix}$$

$$v \in \mathcal{Q} \Leftrightarrow v = \beta_1 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \beta_2 \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ \beta_2 \\ \beta_1 \\ -\beta_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2\alpha_2 \\ \alpha_1 \\ \alpha_3 \\ \alpha_1 \end{pmatrix} = \begin{pmatrix} 0 \\ \beta_2 \\ \beta_1 \\ -\beta_2 \end{pmatrix} \Rightarrow \begin{cases} \alpha_2 = 0 \\ \alpha_1 = \beta_2 \\ \alpha_3 = \beta_1 \\ \alpha_1 = -\beta_2 \end{cases} \begin{cases} \alpha_2 = 0 \\ \alpha_1 = -\beta_2 \\ \alpha_3 = \beta_1 \\ \beta_2 = 0 \end{cases} \begin{cases} \alpha_2 = 0 \\ \alpha_1 = 0 \\ \alpha_3 = \beta_1 \\ \beta_2 = 0 \end{cases}$$

$$\Rightarrow v = 0 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} + 0 \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \beta_1 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \beta_1 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow \mathcal{X}_1 = \text{span}\left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

$$\mathcal{X}_2 : \mathcal{X}_1 \oplus \mathcal{X}_2 = \mathcal{R} \Rightarrow \mathcal{X}_2 = \text{span}\left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

$$\mathcal{X}_3 : \mathcal{X}_1 \oplus \mathcal{X}_3 = \mathcal{Q} \Rightarrow \mathcal{X}_3 = \text{span}\left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \right\}$$

$$\mathcal{X}_4 : \mathcal{X}_1 \oplus \mathcal{X}_2 \oplus \mathcal{X}_3 \oplus \mathcal{X}_4 = \mathbb{R}^4 \Rightarrow \mathcal{X}_4 = \{\emptyset\}$$

$x_A = \begin{pmatrix} 0 \\ -1 \\ 1 \\ 1 \end{pmatrix} \rightarrow$ non è ragg. ma è ~~oss~~ inoss.

$x_B = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} \rightarrow$ è ragg. ~~ma~~ ed oss.

Ex. 5

$$\begin{cases} \dot{x}_1(t) = x_1(t)\left(k - 1 + x_2(t) - x_1^2(t)\right) \\ \dot{x}_2(t) = 1 - x_2(t) - \frac{1}{2} x_1^2(t) \end{cases} \qquad x_B = (0, 1)$$

$$f(x) = \begin{bmatrix} x_1(t)\left(k - 1 + x_2(t) - x_1^2(t)\right) \\ 1 - x_2(t) - 1/2\, x_1^2(t) \end{bmatrix}$$

$f(x_B) = 0 \Rightarrow x_B$ è di eq.

$$\left.\frac{df}{dx}\right|_{x_B} = \left.\begin{bmatrix} (k - 1 + x_2 - x_1^2) + x_1(-2x_1) & x_1 \\ -x_1 & -1 \end{bmatrix}\right|_{x_B} = \begin{bmatrix} k & 0 \\ 0 & -1 \end{bmatrix}$$

$\lambda_1 = -1 \rightarrow Re(\lambda_1) < 0 \qquad \lambda_2 = k$

- Se $k>0 \Rightarrow Re(\lambda_2)>0$, x_B INST
- Se $k<0 \Rightarrow Re(\lambda_2)<0$, x_B LOCALMENTE A.S.
- Se $k=0 \Rightarrow$ CASO CRITICO

Per $k=0$

$$\begin{cases} \dot{x}_1(t) = x_1(t)\left(\cancel{x_2(t)} -1 + x_2(t) - x_1^2(t)\right) \\ \dot{x}_2(t) = 1 - x_2(t) - \frac{1}{2}x_1^2(t) \end{cases}$$

$\alpha, \beta > 0$

Scelgo $V(x) = \frac{\alpha}{2}x_1^2 + \frac{\beta}{2}(x_2-1)^2$ $\quad V(x)>0 \quad V(x_B)=0$

$$\dot{V}(x) = \begin{bmatrix} \alpha x_1 & \beta(x_2-1) \end{bmatrix} \begin{bmatrix} -x_1 + x_1 x_2 - x_1^3 \\ 1 - x_2 - \frac{1}{2}x_1^2 \end{bmatrix}$$

$$= \underbrace{-\alpha x_1^2}_{<0} + \alpha x_1^2 x_2 \underbrace{- \alpha x_1^4}_{<0} + \beta x_2 - \beta x_2^2 - \frac{\beta}{2}x_1^2 x_2 - \beta + \beta x_2 + \frac{\beta}{2}x_1^2$$

$$2x_1^2 x_2 + 2\beta x_2 - \beta x_2^2 - \frac{\beta}{2}x_1^2 x_2 - \beta + \frac{\beta}{2}x_1^2$$

$\xi_2 = x_2 - 1$ $\qquad$ TRASLAZIONE DELLO STATO

$\xi_1 = x_1$

$V(\xi) = \xi_1^2 + \beta\,\xi_2^2$, $\beta > 0$ $\quad V(\xi) > 0$

$$f(\xi) = \begin{bmatrix} \xi_1(\xi_2 - \xi_1^2) \\ -\xi_2 - \frac{1}{2}\xi_1^2 \end{bmatrix}$$

$$\dot{V}(\xi) = \begin{bmatrix} 2\xi_1 & 2\beta\xi_2 \end{bmatrix} \begin{bmatrix} f(\xi) \end{bmatrix} = 2\xi_1^2\xi_2 - 2\xi_1^4 - 2\beta\xi_2^2 - \beta\xi_1^2\xi_2 =$$

$$= (2-\beta)\xi_1^2\xi_2 - 2\xi_1^4 - 2\beta\xi_2^2$$

$\cancel{\begin{cases} 2-\beta<0 \\ -2\beta<0 \end{cases}}$ $\quad$ Se $\beta = 2 \Rightarrow \dot{V}(\xi) = -2\xi_1^4 - 4\xi_2^2 < 0$

$\Rightarrow x_e$ è loc. A.S.