# Università degli Studi dell'Aquila

**Non-Cooperative Networks: Mid-term Evaluation**

Wednesday, November 17th, 2021 – Prof. Guido Proietti

| [redacted] | Last name: .................. First name: ........... ID number: .................. | Points |
|---|---|---|
| EXERCISE 1 | | |
| EXERCISE 2 | | |
| TOTAL | | |

**EXERCISE 1: Multiple-choice questions (20 points)**

**Remark:** Only one choice is correct. Use the enclosed grid to select your choice. A correct answer scores 3 points, while a wrong answer receives a $-1$ penalization. You are allowed to omit an answer. If you wrongly select an answer, just make a circle around the wrong $\times$ (i.e., in the following way $\otimes$) and select through a $\times$ the newly selected answer. A question collecting more than one answer will be considered as omitted. The final score will be given by summing up all the obtained points (0 for a missing answer), and then normalizing to 20.

1. Which of the following claim is <u>false</u> as far as the *Dominant Strategy Equilibrium* is concerned?:
a) if $p_i$ is a cost, it is a strategy combination $s^* = (s_1^*, \ldots, s_N^*)$, such that for each player $i$ and for any possible alternative strategy profile $s = (s_1, \ldots, s_i, \ldots, s_N)$, $p_i(s_1, \ldots, s_i^*, \ldots, s_N) \leq p_i(s_1, \ldots, s_i, \ldots, s_N)$
b) if $p_i$ is a utility, it is a strategy combination $s^* = (s_1^*, \ldots, s_N^*)$, such that for each player $i$ and for any possible alternative strategy profile $s = (s_1, \ldots, s_i, \ldots, s_N)$, $p_i(s_1, \ldots, s_i^*, \ldots, s_N) \geq p_i(s_1, \ldots, s_i, \ldots, s_N)$
*c) if $p_i$ is a cost, it is a strategy combination $s^* = (s_1^*, \ldots, s_N^*)$, such that for each player $i$ and for any possible alternative strategy profile $s = (s_1, \ldots, s_i, \ldots, s_N)$, $p_i(s_1^*, \ldots, s_i^*, \ldots, s_N^*) \leq p_i(s_1, \ldots, s_i, \ldots, s_N)$
d) Dominant Strategy is the best possible response to any strategy of other players

2. Which of the following claim is <u>true</u> as far as the *Nash Equilibrium* (NE) is concerned?
a) It can be shown that there exist games for which finding a NE in mixed strategies is NP-hard
b) Any game with a finite set of players and a finite set of strategies has a NE of pure strategies
c) In the Head and Tail game, it does not exist a NE in mixed startegies
*d) Finding a NE in pure strategies is NP-hard for many games

3. Which of the following claim is <u>false</u> for the Prisoner's Dilemma game:
a) It does admit a Nash equilibrium b) It does admit a dominant strategy equlibrium
c) It has a Price of Anarchy equal to 5 *d) It has a Price of Stability equal to 4/3

4. How the Price of Anarchy is defined for a game in which the social choice function $C$ has to be minimized ($S$ is the set of Nash equilibria)?
a) $\text{PoA} = \inf_{s \in S} \frac{C(s)}{C(\text{OPT})}$ b) $\text{PoA} = \sup_{s \in S} \frac{C(\text{OPT})}{C(s)}$ *c) $\text{PoA} = \sup_{s \in S} \frac{C(s)}{C(\text{OPT})}$ d) $\text{PoA} = \inf_{s \in S} \frac{C(\text{OPT})}{C(s)}$

5. Which of the following claim is <u>false</u> in the Pigou's game:
a) It does admit a Nash equilibrium *b) It does not admit a dominant strategy equlibrium
c) The cost of the optimal flow is 0.75 d) The cost of the Nash flow is 1.

6. Which of the following claim is <u>false</u> as far as the Global Connection Game (GCG) is concerned?
a) A GCG is a potential game *b) The PoA of a GCG with $k$ players is at most $H_k$
c) Finding a best possible NE in a GCG is NP-hard d) A best response for a player in a GCG can be found in polynomial time

7. In a Local Connection Game with $k$ players and building cost $\alpha \geq 0$, which of the following claim is <u>true</u>?
a) A LCG is a potential game b) for $\alpha \leq 2$, the complete graph is a stable solution
c) for $\alpha \geq 1$, the star is an optimal solution *d) Finding a best response for a player in a LCG is NP-hard

8. Which of the following claim is <u>false</u> as far as the Vickrey's Auction is concerned?
a) It satisfies voluntary participation *b) It does not make use of a Clarke payment scheme
c) It is a VCG-mechanism d) It is associated with a single-parameter problem

9. Which of the following corresponds to the definition of the Ackermann function?
a) $A(1,j) = 2^j$ for $j \geq 1$, $A(i,1) = A(i-1,2)$ for $i \geq 2$, $A(i,j) = A(i-1, A(i-1, j-1)$ for $i,j \geq 2$
*b) $A(1,j) = 2^j$ for $j \geq 1$, $A(i,1) = A(i-1,2)$ for $i \geq 2$, $A(i,j) = A(i-1, A(i, j-1)$ for $i,j \geq 2$
c) $A(1,j) = 2^j$ for $j \geq 1$, $A(i,1) = A(i-1,2)$ for $i \geq 2$, $A(i,j) = A(i, A(i, j-1)$ for $i,j \geq 2$
d) $A(1,j) = 2^j$ for $j \geq 1$, $A(i,1) = A(i-1,2)$ for $i \geq 2$, $A(i,j) = A(1, A(i, j-1)$ for $i,j \geq 2$

10. In the one-parameter mechanism for the single-source shortest path tree problem, which payment will receive an edge $e$ belonging to the solution?
a) $p_e = r_e w_e(r) + \int_0^\infty w_e(r_{-e}, z)\, dz$ *b) $p_e = r_e w_e(r) + \int_{r_e}^\infty w_e(r_{-e}, z)\, dz$
c) $p_e = -r_e w_e(r) + \int_{r_e}^\infty w_e(r_{-e}, z)\, dz$ d) $p_e = r_e w_e(r) + \int_0^{r_e} w_e(r_{-e}, z)\, dz$

**Answer Grid**

| | Question | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Choice | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| a | | | | | | | | | | |
| b | | | | | | | | | | |
| c | | | | | | | | | | |
| d | | | | | | | | | | |

**EXERCISE 2: Open question (10 points)**

**Remark:** Select at your choice one out of the following two questions, and address it exhaustively.

1. Describe and analyze the global connection game.
2. Describe and analyze the one-parameter mechanism for the private-edge shortest path tree problem.