# Università degli Studi dell' Aquila 

Non-Cooperative Networks: Mid-term Evaluation

Wednesday, November 6th, 2019 - Prof. Guido Proietti

|  | Last name: | First name: | ID number: | Points |
| :---: | :---: | :---: | :---: | :---: |
| EXERCISE 1 |  |  |  |  |
| EXERCISE 2 |  |  |  |  |
| TOTAL |  |  |  |  |

## EXERCISE 1: Multiple-choice questions (20 points)

Remark: Only one choice is correct. Use the enclosed grid to select your choice. A correct answer scores 3 points, while a wrong answer receives a -1 penalization. You are allowed to omit an answer. If you wrongly select an answer, just make a circle around the wrong $\times$ (i.e., in the following way $\otimes$ ) and select through a $\times$ the newly selected answer. A question collecting more than one answer will be considered as omitted. The final score will be given by summing up all the obtained points ( 0 for a missing answer), and then normalizing to 20 .

1. What does the Nash Theorem state?
*a) Any game with a finite set of players and a finite set of strategies has a NE of mixed strategies
b) Any game with a finite set of players and a finite set of strategies has a NE of pure strategies c) Any game with a finite set of players and a finite set of strategies has at least a NE of mixed strategies d) Any game with any number of players each with a finite set of strategies has a NE of mixed strategies
2. Which of the following claim is true for the Battle of the Sexes game:
a) It admits a single Nash equilibrium
b) It admits two dominant strategy equlibria
*c) It has a Price of Anarchy equal to 1
d) It does not admit any Nash equilibrium
3. Which of the following claim is false for the Prisoner's Dilemma game:
a) It admits a Nash equilibrium b) It admits a dominant strategy equlibrium
c) It has a Price of Anarchy equal to $5 \quad *$ d) It has a Price of Stability equal to 1
4. How the Price of Stability is defined for a game in which the social-choice function $C$ has to be maximized ( $S$ is the set of Nash equilibria)?
*a) $\operatorname{PoS}=\sup _{s \in S} \frac{C(s)}{C(\mathrm{OPT})}$
b) $\operatorname{PoS}=\inf _{s \in S} \frac{C(s)}{C(\text { OPT })}$
c) $\operatorname{PoS}=\sup _{s \in S} \frac{C(\mathrm{OPT})}{C(s)}$
d) $\operatorname{PoS}=\inf _{s \in S} \frac{C(\mathrm{OPT})}{C(s)}$
5. How the Price of Anarchy is defined for a game in which the social choice function $C$ has to be minimized ( $S$ is the set of Nash equilibria)?
*a) $\operatorname{PoA}=\sup _{s \in S} \frac{C(s)}{C(\text { OPT })}$
b) $\operatorname{PoA}=\inf _{s \in S} \frac{C(s)}{C(\text { OPT })}$
c) $\mathrm{PoA}=\sup _{s \in S} \frac{C(\mathrm{OPT})}{C(s)}$
d) $\mathrm{PoA}=\inf _{s \in S} \frac{C(\mathrm{OPT})}{C(s)}$
6. In the global connection game on a graph $G=(V, E, c)$, if we denote by $c_{e}$ (resp., $k_{e}$ ) the cost (resp., the load) of an edge $e \in E$, and by $N(S)$ the network induced by a given strategy profile $S$, which of the following is a potential function?
*a) $\Psi(S)=\sum_{e \in N(S)} c_{e} \cdot\left(1+1 / 2+\ldots 1 / k_{e}\right) \quad$ b) $\Psi(S)=\sum_{e \in N(S)} c_{e} / k_{e}$
c) $\Psi(S)=\sum_{e \in N(S)} c_{e} \quad$ d) $\Psi(S)=\sum_{e \in N(S)} c_{e} \cdot\left(1+2+\ldots+k_{e}\right)$
7. In a local connection game with $k$ players and building cost $\alpha \geq 0$, which of the following claim is true?
a) for $\alpha=3 / 2, \operatorname{PoS}=1$
b) for $\alpha=1$, the clique is the only stable graph
c) $\mathrm{PoA}=O(1)$
*d) $\operatorname{PoS} \leq 4 / 3$
8. In the Malik, Mittal and Gupta algorithm on a graph with $n$ nodes and $m$ edges, which of the following set of operations are performed on the Fibonacci heap?
a) A single make-heap, $O(n)$ insert, $n$ find-min, $O(n)$ delete and $O(m)$ decrease-key
b) A single make-heap, $n$ insert, $O(n)$ find-min, $n$ delete and $O(m)$ decrease-key
${ }^{*}$ c) A single make-heap, $n$ insert, $O(n)$ find-min, $O(n)$ delete and $O(m)$ decrease-key
d) A single make-heap, $n$ insert, $O(n)$ find-min, $O(n)$ delete and $m$ decrease-key
9. In the selfish-edge single-source shortest-path tree problem, which of the following corresponds to the threshold value for an edge $e=(u, v)$ belonging to the solution?
*a) $\Theta_{e}=\min _{f=(x, y) \in C(e)}\left\{d_{G}(s, x)+r(e)+d_{G}(y, v)\right\}-d_{G}(s, u)$
b) $\Theta_{e}=\min _{f=(x, y) \in C(e)}\left\{d_{G}(s, x)+r(e)+d_{G}(y, v)\right\}-d_{G}(s, v)$ c) $\left.\Theta_{e}=\min _{f=(x, y) \in C(e)}\left\{d_{G-e}(s, x)+r(e)+d_{G-e}(y, v)\right\}-d_{G}(s, u) \mathrm{d}\right) \Theta_{e}=\min _{f=(x, y) \in C(e)}\left\{d_{G-e}(s, x)+r(e)+d_{G-e}(y, v)\right\}-$ $d_{G}(s, v)$
10. In the one-parameter mechanism for the single-source shortest path tree problem, which payment will receive an edge $e$ belonging to the solution?
$\begin{array}{ll}*_{\text {a) }} p_{e}=r_{e} w_{e}(r)+\int_{r_{e}}^{\infty} w_{e}\left(r_{-e}, z\right) d z & \text { b) } p_{e}=r_{e} w_{e}(r)+\int_{0}^{\infty} w_{e}\left(r_{-e}, z\right) d z \\ \text { c) } p_{e}=-r_{e} w_{e}(r)+\int_{r_{e}}^{\infty} w_{e}\left(r_{-e}, z\right) d z & \text { d) } p_{e}=r_{e} w_{e}(r)+\int_{0}^{r_{e}} w_{e}\left(r_{-e}, z\right) d z\end{array}$
c) $p_{e}=-r_{e} w_{e}(r)+\int_{r_{e}}^{\infty} w_{e}\left(r_{-e}, z\right) d z \quad$ d) $p_{e}=r_{e} w_{e}(r)+\int_{0}^{r_{e}} w_{e}\left(r_{-e}, z\right) d z$

Answer Grid

|  | Question |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Choice | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| a |  |  |  |  |  |  |  |  |  |  |
| b |  |  |  |  |  |  |  |  |  |  |
| c |  |  |  |  |  |  |  |  |  |  |
| d |  |  |  |  |  |  |  |  |  |  |

## EXERCISE 2: Open question (10 points)

Remark: Select at your choice one out of the following two questions, and address it exhaustively.

1. Describe and analyze the global connection game.
2. Describe and analyze the one-parameter mechanism for the private-edge SPT problem.
