## UNIVERSITÀ DEGLI STUDI DELL'AQUILA Non-Cooperative Networks: Mid-term Evaluation

Wednesday, November 6th, 2019 – Prof. Guido Proietti

	Last name: F	First name:	ID number:	Points
EXERCISE 1				
EXERCISE 2				
TOTAL				

## EXERCISE 1: Multiple-choice questions (20 points)

**Remark:** Only one choice is correct. Use the enclosed grid to select your choice. A correct answer scores 3 points, while a wrong answer receives a -1 penalization. You are allowed to omit an answer. If you wrongly select an answer, just make a circle around the wrong  $\times$  (i.e., in the following way  $\otimes$ ) and select through a  $\times$  the newly selected answer. A question collecting more than one answer will be considered as omitted. The final score will be given by summing up all the obtained points (0 for a missing answer), and then normalizing to 20.

- 1. What does the Nash Theorem state?
  - \*a) Any game with a finite set of players and a finite set of strategies has a NE of mixed strategies

b) Any game with a finite set of players and a finite set of strategies has a NE of pure strategies c) Any game with a finite set of players and a finite set of strategies has at least a NE of mixed strategies d) Any game with any number of players each with a finite set of strategies has a NE of mixed strategies

- 2. Which of the following claim is true for the Battle of the Sexes game:
  a) It admits a single Nash equilibrium
  b) It admits two dominant strategy equilibria
  \*c) It has a Price of Anarchy equal to 1
  d) It does not admit any Nash equilibrium
- 3. Which of the following claim is false for the Prisoner's Dilemma game:
  a) It admits a Nash equilibrium b) It admits a dominant strategy equilibrium
  c) It has a Price of Anarchy equal to 5 \*d) It has a Price of Stability equal to 1
- 4. How the Price of Stability is defined for a game in which the social-choice function C has to be maximized (S is the set of Nash equilibria)?

\*a) 
$$\operatorname{PoS} = \sup_{s \in S} \frac{C(s)}{C(OPT)}$$
 b)  $\operatorname{PoS} = \inf_{s \in S} \frac{C(s)}{C(OPT)}$  c)  $\operatorname{PoS} = \sup_{s \in S} \frac{C(OPT)}{C(s)}$  d)  $\operatorname{PoS} = \inf_{s \in S} \frac{C(OPT)}{C(s)}$ 

5. How the Price of Anarchy is defined for a game in which the social choice function C has to be minimized (S is the set of Nash equilibria)?

\*a) 
$$\operatorname{PoA} = \sup_{s \in S} \frac{C(s)}{C(\operatorname{OPT})}$$
 b)  $\operatorname{PoA} = \inf_{s \in S} \frac{C(s)}{C(\operatorname{OPT})}$  c)  $\operatorname{PoA} = \sup_{s \in S} \frac{C(\operatorname{OPT})}{C(s)}$  d)  $\operatorname{PoA} = \inf_{s \in S} \frac{C(\operatorname{OPT})}{C(s)}$ 

6. In the global connection game on a graph G = (V, E, c), if we denote by  $c_e$  (resp.,  $k_e$ ) the cost (resp., the load) of an edge  $e \in E$ , and by N(S) the network induced by a given strategy profile S, which of the following is a potential function? \*a)  $\Psi(S) = \sum_{e \in N(S)} c_e \cdot (1 + 1/2 + \dots 1/k_e)$  b)  $\Psi(S) = \sum_{e \in N(S)} c_e/k_e$ c)  $\Psi(S) = \sum_{e \in N(S)} c_e$  d)  $\Psi(S) = \sum_{e \in N(S)} c_e \cdot (1 + 2 + \dots + k_e)$ 

- 7. In a local connection game with k players and building cost  $\alpha \ge 0$ , which of the following claim is true?
- a) for  $\alpha = 3/2$ , PoS = 1 b) for  $\alpha = 1$ , the clique is the only stable graph c) PoA = O(1) \*d) PoS  $\leq 4/3$
- 8. In the Malik, Mittal and Gupta algorithm on a graph with n nodes and m edges, which of the following set of operations are performed on the Fibonacci heap?
  - a) A single make-heap, O(n) insert, n find-min, O(n) delete and O(m) decrease-key
  - b) A single make-heap, n insert,  ${\cal O}(n)$  find-min, n delete and  ${\cal O}(m)$  decrease-key
  - \*c) A single make-heap, n insert, O(n) find-min, O(n) delete and O(m) decrease-key
  - d) A single make-heap, n insert,<br/>  ${\cal O}(n)$  find-min,  ${\cal O}(n)$  delete and<br/> m decrease-key
- 9. In the selfish-edge single-source shortest-path tree problem, which of the following corresponds to the threshold value for an edge e = (u, v) belonging to the solution?
  - \*a)  $\Theta_e = \min_{f=(x,y)\in C(e)} \{ d_G(s,x) + r(e) + d_G(y,v) \} d_G(s,u) \text{ b) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_G(s,x) + r(e) + d_G(y,v) \} d_G(s,v) \text{ c) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,u) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \min_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \max_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \max_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \max_{f=(x,y)\in C(e)} \{ d_{G-e}(s,x) + r(e) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \max_{f=(x,y)\in C(e)} \{ d_{G-e}(s,v) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \max_{f=(x,y)\in C(e)} \{ d_{G-e}(s,v) + d_{G-e}(y,v) \} d_G(s,v) \text{ d) } \Theta_e = \max_{f=(x,y)\in C(e)} \{ d_{G-e}(s,v) + d_{G-e}(y,v) \} d_G(s,v) \} + (d_{G-e}(s,v) + d_{G-e}(y,v) \} + (d_{G-e}(s,v) + d_{G-e}(y,v) \} + (d_{G-e}(y,v) \} + (d_{G-e}(y,v) \} + (d_{G-e}(y,$
- 10. In the one-parameter mechanism for the single-source shortest path tree problem, which payment will receive an edge e belonging to the solution?

\*a) 
$$p_e = r_e w_e(r) + \int_{r_e}^{\infty} w_e(r_{-e}, z) dz$$
 b)  $p_e = r_e w_e(r) + \int_0^{\infty} w_e(r_{-e}, z) dz$   
c)  $p_e = -r_e w_e(r) + \int_{r_e}^{\infty} w_e(r_{-e}, z) dz$  d)  $p_e = r_e w_e(r) + \int_0^{r_e} w_e(r_{-e}, z) dz$ 

Answer Grid

	Question									
Choice	1	2	3	4	5	6	7	8	9	10
a										
b										
с										
d										

## EXERCISE 2: Open question (10 points)

Remark: Select at your choice one out of the following two questions, and address it exhaustively.

- 1. Describe and analyze the global connection game.
- 2. Describe and analyze the one-parameter mechanism for the private-edge SPT problem.