Università degli Studi dell'Aquila

Non-Cooperative Networks: Mid-term Evaluation

Thursday, November 19th, 2020 – Prof. Guido Proietti

[redacted]	Last name: .... First name: .... ID number:	Points
EXERCISE 1		
EXERCISE 2		
TOTAL		

EXERCISE 1: Multiple-choice questions (20 points)
Remark: Only one choice is correct. Use the enclosed grid to select your choice. A correct answer scores 3 points, while a wrong answer receives a -1 penalization. You are allowed to omit an answer. If you wrongly select an answer, just make a circle around the wrong $\times$ (i.e., in the following way $\otimes$) and select through a $\times$ the newly selected answer. A question collecting more than one answer will be considered as omitted. The final score will be given by summing up all the obtained points (0 for a missing answer), and then normalizing to 20.

1. A *Dominant Strategy Equilibrium* is a strategy combination $s^* = (s_1^*, \ldots, s_N^*)$, such that (assume p_i is a cost):
   a) there exists a player i and an alternative strategy profile $s = (s_1, \ldots, s_i, \ldots, s_N)$, such that $p_i(s_1, \ldots, s_i^*, \ldots, s_N) \geq p_i(s_1, \ldots, s_i, \ldots, s_N)$
   b) there exist no player i and no alternative strategy profile $s = (s_1, \ldots, s_i, \ldots, s_N)$, such that $p_i(s_1, \ldots, s_i^*, \ldots, s_N) \leq p_i(s_1, \ldots, s_i, \ldots, s_N)$
   c) for each player i and for any possible alternative strategy profile $s = (s_1, \ldots, s_i, \ldots, s_N)$, $p_i(s_1^*, \ldots, s_i^*, \ldots, s_N^*) \geq p_i(s_1, \ldots, s_i, \ldots, s_N)$
   *d) for each player i and for any possible alternative strategy profile $s = (s_1, \ldots, s_i, \ldots, s_N)$, $p_i(s_1, \ldots, s_i^*, \ldots, s_N) \leq p_i(s_1, \ldots, s_i, \ldots, s_N)$
2. A *Nash Equilibrium* is a strategy combination $s^* = (s_1^*, \ldots, s_N^*)$, such that (assume p_i is a utility):
   a) there exists a player i and an alternative strategy profile $s = (s_1, \ldots, s_i, \ldots, s_N)$, such that $p_i(s_1, \ldots, s_i^*, \ldots, s_N) \leq p_i(s_1, \ldots, s_i, \ldots, s_N)$
   b) for each player i and for any possible alternative strategy profile $s = (s_1, \ldots, s_i, \ldots, s_N)$, $p_i(s_1, \ldots, s_i^*, \ldots, s_N) \geq p_i(s_1, \ldots, s_i, \ldots, s_N)$
   *c) for each player i and for any alternative strategy s_i of i, $p_i(s_1^*, \ldots, s_i^*, \ldots, s_N^*) \geq p_i(s_1^*, \ldots, s_i, \ldots, s_N^*)$
   d) there exist no player i and no alternative strategy profile $s = (s_1, \ldots, s_i, \ldots, s_N)$, such that $p_i(s_1, \ldots, s_i^*, \ldots, s_N) \leq p_i(s_1, \ldots, s_i, \ldots, s_N)$
3. Which of the following claim is true for the Prisoner's Dilemma game:
   a) It admits two Nash equilibria b) It does not admit a dominant strategy equlibrium
   c) It has a Price of Anarchy equal to 4/3 *d) It has a Price of Stability equal to 5
4. How the Price of Anarchy is defined for a game in which the social choice function C has to be minimized (S is the set of Nash equilibria)?
   a) $\text{PoA} = \inf_{s \in S} \frac{C(s)}{C(\text{OPT})}$ *b) $\text{PoA} = \sup_{s \in S} \frac{C(s)}{C(\text{OPT})}$ c) $\text{PoA} = \sup_{s \in S} \frac{C(\text{OPT})}{C(s)}$ d) $\text{PoA} = \inf_{s \in S} \frac{C(\text{OPT})}{C(s)}$
5. The Beckmann theorem states that under certain assumptions the Nash flow of (G, s, r, l) exists and is unique, and is equal to the optimal min-cost flow of the instance:
   *a) $(G, s, r, \lambda(x) = [\int_0^x l(t)\, dt]/x)$ b) $(G, s, r, \lambda(x) = \int_0^x l(t)\, dt)$
   c) $(G, s, r, \lambda(x) = [\int_0^{+\infty} l(t)\, dt]/x)$ d) $(G, s, r, \lambda(x) = [\int l(t)\, dt]/x)$
6. In the global connection game on a graph $G = (V, E, c)$, if we denote by c_e (resp., k_e) the cost (resp., the load) of an edge $e \in E$, and by $N(S)$ the network induced by a given strategy profile S, which of the following is a potential function?
   a) $\Psi(S) = \sum_{e \in N(S)} c_e / k_e$ *b) $\Psi(S) = \sum_{e \in N(S)} c_e \cdot (1 + 1/2 + \ldots 1/k_e)$
   c) $\Psi(S) = \sum_{e \in N(S)} c_e$ d) $\Psi(S) = \sum_{e \in N(S)} c_e \cdot (1 + 2 + \ldots + k_e)$
7. In a local connection game with k players and building cost $\alpha \geq 0$, which of the following claim is false?
   a) for $\alpha = 1$, the clique and the star are stable graphs b) $\text{PoA} \leq 6\sqrt{\alpha} + 3$
   * c) for $\alpha \geq 1$, the star is an optimal solution d) $\text{PoS} \leq 4/3$
8. Which of the following is a Clarke payment scheme?
   a) $p_i(g(r)) = \sum_{j \neq i} v_j(r_j, g(r_{-i})) - \sum_{j \neq i} v_j(r_j, g(r))$ b) $p_i(g(r)) = -\sum_{j \neq i} v_j(r_j, g(r)) + \sum_j v_j(r_j, g(r))$
   *c) $p_i(g(r)) = -\sum_{j \neq i} v_j(r_j, g(r_{-i})) + \sum_{j \neq i} v_j(r_j, g(r))$ d) $p_i(g(r)) = -\sum_j v_j(r_j, g(r_{-i})) + \sum_{j \neq i} v_j(r_j, g(r))$
9. Which of the following corresponds to the definition of the Ackermann function?
   a) $A(1, j) = 2^j$ for $j \geq 1$, $A(i, 1) = A(i-1, 2)$ for $i \geq 2$, $A(i, j) = A(i-1, A(i-1, j-1)$ for $i, j \geq 2$
   b) $A(1, j) = 2^j$ for $j \geq 1$, $A(i, 1) = A(i-1, 2)$ for $i \geq 2$, $A(i, j) = A(i, A(i, j-1)$ for $i, j \geq 2$
   c) $A(1, j) = 2^j$ for $j \geq 1$, $A(i, 1) = A(i-1, 2)$ for $i \geq 2$, $A(i, j) = A(1, A(i, j-1)$ for $i, j \geq 2$
   *d) $A(1, j) = 2^j$ for $j \geq 1$, $A(i, 1) = A(i-1, 2)$ for $i \geq 2$, $A(i, j) = A(i-1, A(i, j-1)$ for $i, j \geq 2$
10. In the Malik, Mittal and Gupta algorithm for the selfish-edge shortest path problem, which of the following keys is associated with a node y in the priority queue when an edge e of a graph $G = (V, E)$ is considered?
   a) $k(y) = \min_{(x,y) \in E, x \in N_s(e)} \{d_G(s, x) + r(x, y) + d_G(y, z)\}$ b) $k(y) = \max_{(x,y) \in E, x \in M_s(e)} \{d_G(s, x) + r(x, y) + d_G(y, z)\}$
   c) $k(y) = \min_{(x,y) \in E, x \in M_s(e)} \{d_G(s, x) + 1 + d_G(y, z)\}$ *d) $k(y) = \min_{(x,y) \in E, x \in M_s(e)} \{d_G(s, x) + r(x, y) + d_G(y, z)\}$

Answer Grid

	Question									
Choice	1	2	3	4	5	6	7	8	9	10
a										
b										
c										
d										

EXERCISE 2: Open question (10 points)
Remark: Select at your choice one out of the following two questions, and address it exhaustively.

1. Describe and analyze the selfish routing game.
2. Describe and analyze the VCG-mechanism for the single-edge single-pair shortest path problem.