Laurea Magistrale in Informatica
Formal Methods - Rewriting (2010-2011)

Questions for Intermediate Exam

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1. Let $\mathcal{A} = \langle \{a, b, c\}, \longrightarrow \rangle$ be an ARS where $\longrightarrow$ is defined as follows:

$$\begin{array}{lcl} a & \longrightarrow & b \\ b & \longrightarrow & a \\ b & \longrightarrow & c \\ c & \longrightarrow & b \end{array}$$

Show whether $\longrightarrow$ is normalizing, noetherian, locally confluent or confluent.

2. Prove that an ARS $\langle A, \longrightarrow \rangle$ is confluent if there exists a confluent ARS $\langle A, \longrightarrow_1 \rangle$ such that $\longrightarrow \subseteq \stackrel{*}{\longrightarrow}_1 \subseteq \stackrel{*}{\longrightarrow}$.

3. Compute the most general unifier (if it exists) of the following pairs of terms:
i) $t_1 = f(g(x, y), x)$ and $t_2 = f(x', y')$;
ii) $t_1 = g(x, x)$ and $t_2 = g(h(x'), h(y'))$;
iii) $t_1 = f(x, x)$ and $t_2 = f(x', g(y', x'))$.

4. Given the terms $t_1 = g(x, x)$ and $t_2 = g(g(x', y'), z')$, say if t_1 can be syntactically unified with t_2 and subterms of t_2, and give the most general unifiers (if they exist).

5. Consider the following TRS R over a signature $\Sigma = \{a, f, g, h\}$:

$$\begin{array}{rcl} g(a, x) & \rightarrow & x \\ g(h(x), y) & \rightarrow & h(g(x, y)) \\ f(a) & \rightarrow & a \\ f(h(a)) & \rightarrow & h(a) \\ f(h(h(x))) & \rightarrow & g(f(x), f(h(x))) \end{array}$$

i) Given the term $t = g(h(h(z)), f(h(f(a))))$, apply all the possible reduction steps from t in R by showing the rule applied, the position of the redex and the matching substitution for each reduction step.
ii) Give a reduction ordering on terms such that R is terminating with respect to such an ordering. Motivate your answer.

6. Consider the following TRS R:

$$\begin{array}{rcl} k(h(x, y)) & \rightarrow & f(x, y) \\ f(k(x), y) & \rightarrow & g(k(f(x, y)), f(x, y)) \\ h(f(x, z), y) & \rightarrow & f(h(x, y), h(y, z)) \end{array}$$

Give a reduction ordering on terms such that R is terminating with respect to such an ordering. Motivate your answer.