Laurea Magistrale in Informatica
Formal Methods - Rewriting (2010-2011)

Intermediate Written Exam

December 2nd, 2010

1. Given the signature $\Sigma = \{a, f, g, h\}$ and variables $x, y, x', y' \in V$, compute the most general unifier (if it exists) of the following pairs of terms:
i) $t_1 = g(h(x), g(x, y))$ and $t_2 = g(x', g(x', x'))$;
ii) $t_1 = f(x, h(y), h(x))$ and $t_2 = f(a, x', h(y'))$;
iii) $t_1 = f(x, g(x, y), h(y))$ and $t_2 = f(h(x'), g(h(y'), y'), h(a))$.

2. Given the terms $t_1 = f(g(x, y), z)$ and $t_2 = f(f(x', y'), z')$, say if t_1 can be syntactically unified with t_2 and subterms of t_2, and give the most general unifiers (if they exist).

3. Consider the following TRS R on the signature $\Sigma = \{a, f, g, h, k\}$:

$$\begin{array}{rcl} g(a, x) & \to & a \\ g(h(x), y) & \to & f(k(y), g(x, y)) \\ k(a) & \to & a \end{array}$$

i) Give a reduction ordering on terms such that R is terminating with respect to such an ordering. Show the formal steps that justify your answer.
ii) Reduce the term $t = g(h(h(a)), a)$ to normal form in R, by applying all the possible reduction steps from t in R and showing the rule applied, the position of the redex and the matching substitution for each reduction step.

4. Consider the following TRS R on the signature $\Sigma = \{a, b, f, g, k\}$:

$$\begin{array}{rcl} k(a) & \to & b \\ k(f(x)) & \to & x \\ g(x, b) & \to & x \\ g(x, a) & \to & f(x) \\ g(x, f(y)) & \to & f(g(x, y)) \end{array}$$

Give a reduction ordering on terms such that R is terminating with respect to such an ordering. Show the formal steps that justify your answer.

5. Given an ARS $\mathcal{A} = \langle A, \longrightarrow \rangle$, the *unique normal form* property (UN for short) can be defined as follows: for all $a, b \in A$ if $a \stackrel{*}{\longleftrightarrow} b$ and both a and b are normal forms, then $a \equiv b$. Prove that if an ARS $\mathcal{A}$ is UN and WN, then $\mathcal{A}$ is confluent.