Course on Formal Methods 2009-2010

Rewriting

10 February 2011

Exercise 1. Let R be the following rewrite system on the signature $\Sigma = \{a, f, k\}$:

$$
\begin{aligned}
f(k(x), x) &\rightarrow a \\
f(x, k(x)) &\rightarrow a \\
k(f(x, y)) &\rightarrow f(k(y), k(x)) \\
f(x, a) &\rightarrow x \\
f(a, x) &\rightarrow x
\end{aligned}
$$

i) Give an *ordering on terms* such that the rewrite system R be terminating with respect to such a term ordering.

ii) Compute at least six of the critical pairs derived during the completion of R with respect to the term ordering given in i) above, by applying the following strategy: first compute all critical pairs between the rules in R and next compute the possible critical pairs between the rules derived from the previously computed critical pairs.

Exercise 2. Let R be the following rewrite system describing an equational theory E on the signature $\Sigma = \{a, f, g, h\}$:

$$
\begin{aligned}
f(a, x) &\rightarrow a \\
f(x, a) &\rightarrow x \\
f(h(x), h(y)) &\rightarrow f(x, y) \\
g(a, y) &\rightarrow y \\
g(h(x), y) &\rightarrow h(g(x, y))
\end{aligned}
$$

i) Give an *ordering on terms* such that R be terminating with respect to such a term ordering.

ii) Check that R is confluent.

iii) Solve modulo E the equation $f(h(x), y) = g(x, h(y))$ by applying the E-unification algorithm based on normal and basic narrowing. Give the derivation tree with all the narrowing steps of the first level of the tree, plus all possible normalization steps.