Course on Formal Methods 2010-2011

Rewriting

24 February 2011

Exercise 1. Let R be the following trs on a signature $\Sigma = \{a, b, f, h\}$:

$$
\begin{aligned}
h(x, f(y, z)) &\rightarrow f(h(x, y), h(x, z)) \\
h(x, x) &\rightarrow x \\
h(x, a) &\rightarrow a \\
h(x, b) &\rightarrow x \\
f(x, a) &\rightarrow x \\
f(x, x) &\rightarrow a
\end{aligned}
$$

i) Give an *ordering on terms* such that R be terminating with respect to such a term ordering.

ii) Compute at least six of the critical pairs derived during the completion of R with respect to the term ordering given in i) above, by applying the following strategy: first compute all critical pairs between the rules in R and next compute the possible critical pairs between the rules derived from the previously computed critical pairs.

Exercise 2. Let R be the following trs describing an equational theory E on the signature $\Sigma = \{a, f, g, h, k\}$:

$$
\begin{aligned}
f(x, a) &\rightarrow x \\
f(x, k(y)) &\rightarrow k(f(x, y)) \\
h(x, a) &\rightarrow a \\
h(x, k(y)) &\rightarrow f(h(x, y), x) \\
g(a) &\rightarrow k(a) \\
g(k(x)) &\rightarrow h(k(x), g(x))
\end{aligned}
$$

i) Give an *ordering on terms* such that R be terminating with respect to such a term ordering.

ii) Check that R is confluent.

iii) Solve modulo E the equation $g(x) = h(x, y)$ by applying the E-unification algorithm based on normal and basic narrowing. Give the derivation tree with all the narrowing steps of the first level of the tree, plus all possible normalization steps.