

Course on Formal Methods 2010-2011

Rewriting

27 June 2011

Exercise 1. Let R be the following trs on a signature $\Sigma = \{a, f, k\}$:

$$\begin{aligned}f(x, k(x)) &\rightarrow a \\f(k(x), x) &\rightarrow a \\f(x, f(y, k(x))) &\rightarrow y \\k(f(x, y)) &\rightarrow f(k(x), k(y))\end{aligned}$$

- i) Give an *ordering on terms* such that R be terminating with respect to such a term ordering.
- ii) Compute at least six of the critical pairs derived during the completion of R with respect to the term ordering given in i) above, by applying the following strategy: first compute all critical pairs between the rules in R and next compute the possible critical pairs between the rules derived from the previously computed critical pairs.

Exercise 2. Let R be the following trs describing an equational theory E on the signature $\Sigma = \{a, f, g, h, k\}$:

$$\begin{aligned}f(x, a) &\rightarrow x \\f(h(x), h(y)) &\rightarrow f(x, y) \\g(x, a) &\rightarrow x \\g(x, h(y)) &\rightarrow h(g(x, y)) \\k(x, a) &\rightarrow a \\k(x, h(y)) &\rightarrow g(k(x, y), x)\end{aligned}$$

- i) Give an *ordering on terms* such that R be terminating with respect to such a term ordering.
- ii) Check that R is confluent.
- iii) Solve modulo E the equation $k(h(x), y) = f(x, h(y))$ by applying the E-unification algorithm based on normal and basic narrowing. Give the derivation tree with all the narrowing steps of the first level of the tree and half of the second level, plus all possible normalization steps.