Laurea Magistrale in Informatica
Formal Methods (2011-2012)

24 September 2012

Rewriting

1. Let R be the following trs on a signature $\Sigma = \{a, b, f, g\}$:

$$
\begin{array}{rcl}
f(x, a) & \rightarrow & x \\
f(a, x) & \rightarrow & x \\
f(x, x) & \rightarrow & x \\
g(x, g(b, y)) & \rightarrow & g(x, y) \\
g(a, f(g(b, x), x)) & \rightarrow & g(a, x)
\end{array}
$$

i) Give a *reduction ordering on terms* such that R be terminating with respect to such an ordering. Show the formal steps that justify your answer.
ii) Compute the critical pairs derived during the completion of R with respect to the term ordering given in i) above. Discuss whether R is locally confluent and hence canonical.

2. Let R be the following trs describing an equational theory E on the signature $\Sigma = \{a, f, g, h\}$:

$$
\begin{array}{rcl}
g(x, f(y)) & \rightarrow & h(g(x, y), x) \\
g(x, a) & \rightarrow & a \\
h(x, f(a)) & \rightarrow & f(x)
\end{array}
$$

i) Give a *reduction ordering on terms* such that R be terminating with respect to such an ordering. Show the formal steps that justify your answer.
ii) Check that R is confluent.
iii) Solve modulo E the equation $g(f(x), y) = h(f(x), y)$ by applying the E-unification algorithm based on normal and basic narrowing. Give the derivation tree with all the narrowing steps of the first level of the tree and half of the second level, plus all possible normalization steps.

Logic & Theorem Proving

1. Given the formula $f = ((x \Rightarrow \neg y) \wedge z) \Rightarrow ((x \wedge y) \vee \neg z)$, give (*if they exist*) two assignments to variables x, y, z that make the formula true and two other assignments that make it false.
Then, transform the formula f into CNF.

2. Prove $(A \vee B) \Rightarrow \neg((\neg A) \wedge (\neg B))$ using natural deduction *by indicating the rule applied at each step.*

3. Prove $((A \Rightarrow C) \wedge (B \Rightarrow D)) \Rightarrow ((A \vee B) \Rightarrow (C \vee D))$ using natural deduction *by indicating the rule applied at each step.*

4. Prove $(\forall x. \forall y. (P\,x\,y) \Rightarrow \neg(P\,y\,x)) \Rightarrow (\forall x. \neg(P\,x\,x))$ using natural deduction *by indicating the rule applied at each step.*

5. Given the λ-expression $M = (\lambda x.\, x(\lambda y.\, x\,y))(\lambda z.\, z\,y\,a)$, mark each variable occurrence in M as binding, bound or free. For each bound occurrence, indicate the corresponding binding occurrence. Then, reduce M to β-normal form *by underlining the redex at each step*. The student can choose either an innermost or outermost reduction strategy.

6. Let $\Sigma = \{f : \sigma \rightarrow \rho,\ g : \rho \rightarrow \sigma \rightarrow \tau,\ h : \sigma \rightarrow \tau \rightarrow \sigma\}$ and
$\Gamma = \{x : \sigma,\ y : \tau\}$.
Derive a type judgement for the term $\lambda x^\sigma.\lambda y^\tau.\, g\,(f\,x)\,(h\,x\,y)$ *by indicating the rule applied at each step.*