Laurea Magistrale in Informatica
Formal Methods (2015-2016)

20 June 2016

Second partial written exam

Rewriting

1. Let R be the following trs on a signature $\Sigma = \{a, f, g\}$:

$$
\begin{array}{rcl}
f(a, x) & \rightarrow & x \\
f(x, a) & \rightarrow & x \\
f(x, g(y, z)) & \rightarrow & g(f(x, y), f(x, z)) \\
f(x, f(y, z)) & \rightarrow & f(f(x, y), z)
\end{array}
$$

Compute four of the critical pairs derived during the completion of R with respect to an rpo based on $f > g$, by applying the following strategy: first compute all critical pairs between the rules in R and next compute the possible critical pairs between the rules derived from the previously computed critical pairs.

2. Let R be the following canonical trs describing an equational theory E on the signature $\Sigma = \{a, f, g, h\}$:

$$
\begin{array}{rcl}
f(x, a) & \rightarrow & x \\
f(x, h(y)) & \rightarrow & h(f(x, y)) \\
g(x, a) & \rightarrow & x \\
g(a, x) & \rightarrow & a \\
g(h(x), h(y)) & \rightarrow & g(x, y)
\end{array}
$$

Solve modulo E the equation $f(h(x), y) = g(x, h(y))$ by applying the E-unification algorithm based on normal and basic narrowing. Give the derivation tree with *all* the narrowing steps of the first level of the tree, plus all possible normalization steps, and indicate the basic positions for the second level.

Logic & Theorem Proving

1. Let the formula $f = (x \Rightarrow \neg y) \Rightarrow ((y \Rightarrow z) \Rightarrow \neg x)$. Show that f is satisfiable. Give (*if they exist*) two assignments to variables x, y, z that make f true and two assignments that make f false *without generating the truth table*.
Transform the formula f into CNF.

2. Let the formula $H = (\neg A \vee B) \Rightarrow (\neg(A \wedge B) \Rightarrow \neg A)$. Show whether or not H is a tautology by applying i) the axioms for deriving the CNF and ii) the rules of natural deduction.

3. Prove $(A \Rightarrow B) \Rightarrow ((B \Rightarrow A) \Rightarrow ((A \vee B) \Rightarrow (A \wedge B)))$ using natural deduction *by indicating the rule applied at each step*.

4. Prove $(\exists x.\, \neg(P\, x\, x)) \Rightarrow \neg(\forall x.\, (P\, x\, x))$ using natural deduction *by indicating the rule applied at each step*.

5. Given the λ-expression $M = (\lambda x.x(\lambda z.xz)y)(\lambda yw.ywz)$, mark each variable occurrence in M as binding, bound or free. For each bound occurrence, indicate the corresponding binding occurrence. Then, reduce M to β-normal form *by underlining the redex at each step*. The student can choose either an innermost or outermost reduction strategy.

6. Let $\Sigma = \{f : \sigma \rightarrow (\rho \rightarrow \sigma) \rightarrow \sigma \rightarrow \tau,\ g : \sigma \rightarrow \tau \rightarrow \rho \rightarrow \sigma,\ h : \rho \rightarrow \sigma\}$ and $\Gamma = \{x : \sigma,\ y : \rho,\ z : \tau\}$.
Derive a type judgement for the term $\lambda x^{\sigma}.(\lambda z^{\tau}.\, (f\, x)(g\, (h\, y)\, z)\, x)$
by indicating the rule applied at each step.