# Laurea Magistrale in Informatica
# Formal Methods (2013-2014)

## Midterm Exam - November 25th, 2013

**1**. Let $\mathcal{A} = \langle\{a,b,c,d\}, \longrightarrow\rangle$ and $\mathcal{A}_1 = \langle\{a',b',c',d',e'\}, \longrightarrow_1\rangle$ be two ARSs, where $\longrightarrow$ and $\longrightarrow_1$ are defined as follows:

$$\begin{array}{lcl} a & \longrightarrow & b \\ a & \longrightarrow & c \\ b & \longrightarrow & b \\ b & \longrightarrow & d \\ c & \longrightarrow & a \end{array} \qquad\qquad \begin{array}{lcl} a' & \longrightarrow_1 & b' \\ b' & \longrightarrow_1 & b' \\ b' & \longrightarrow_1 & c' \\ b' & \longrightarrow_1 & d' \\ c' & \longrightarrow_1 & a' \\ c' & \longrightarrow_1 & e' \end{array}$$

Show whether $\longrightarrow$ and $\longrightarrow_1$ are weakly normalizing, noetherian, locally confluent, confluent or UN (uniqueness of normal forms). Justify your answers.

**2**. Given the signature $\Sigma = \{c, f, g, h\}$ and variables $x, y, z, w \in V$, compute the (idempotent) most general unifier (if it exists) for each pair of terms:
i) $t_1 = f(h(c,x), h(x,y))$ and $t_2 = f(h(x,c), z)$;
ii) $t_1 = g(g(x,y,x), f(x,c), h(y,y))$ and $t_2 = g(g(z,c,w), z, w)$;
iii) $t_1 = g(x, h(f(y,y),c), g(x,y,y))$ and $t_2 = g(h(z,z), h(w,z), g(w,c,z))$;
iv) $t_1 = g(f(c,x), x, f(y,c))$ and $t_2 = g(z, h(c,w), f(z,w))$.

**3**. Consider the following trs $R$ on the signature $\Sigma = \{a, b, f, g, h, k\}$:

$$\begin{array}{rcl} f(x,b) & \to & x \\ f(x,k(y)) & \to & k(f(x,y)) \end{array} \qquad\qquad \begin{array}{rcl} g(b,x) & \to & a \\ g(k(x),a) & \to & a \\ g(k(x),h(y,z)) & \to & h(y,g(x,z)) \end{array}$$

i) Give a *reduction ordering on terms* such that $R$ is terminating with respect to such an ordering. Show the formal steps that justify your answer.
ii) Reduce the term $t = g(f(k(b),k(b)), h(b,h(k(b),a)))$ to normal form in $R$, by applying all possible reductions from $t$ in $R$ using an *innermost* strategy in case of more redexes (i.e. apply the reduction on the redex at the deepest position). Show the rule applied, the position of the redex and the match for each reduction step.
iii) Given the term $s = g(f(k(z),b), h(w,h(w,a)))$ where $z, w \in V$, how many reduction steps can be applied in $R$ starting from $s$?
iv) Does $R$ have any critical pairs? If it does, check if they are convergent in $R$.

**4**. Consider the following trs $R$ on the signature $\Sigma = \{a, f, g\}$:

$$\begin{array}{rcl} f(x,a) & \to & x \\ f(g(x),x) & \to & a \\ g(f(x,y)) & \to & f(g(x),g(y)) \end{array}$$

i) Give a *reduction ordering on terms* such that $R$ is terminating with respect to such an ordering. Show the formal steps that justify your answer.
ii) Compute all critical pairs of $R$ and show if they are convergent in $R$.