Analisi Matematica 1 - Esercizi - 9/2/2007

(1) Dimostrare per induzione che:

1. $\sum_{k=1}^{n} \left(\frac{2}{5}\right)^k = \frac{2}{3}\left[1 - \left(\frac{2}{5}\right)^n\right]$, $n = 1, 2, \dots$
2. $\sum_{k=1}^{n} \frac{3k}{2k+1} \leq 2n, \;\; \forall\, n \in \mathbb{N},\, n \geq 1$
3. $2^{n-1} > 6n + 1, \;\; \forall\, n \in \mathbb{N},\, n \geq 7$
4. $3^n > n + 1$ per ogni intero $n \geq 1$
5. $\Pi_{k=1}^{n} 3^{2k-1} = 3^{n^2}$

(2) Calcolare il limite per $n \to +\infty$ delle seguenti successioni:

1. $n^2 \sin\left(n\frac{\pi}{2}\right)$
2. $\dfrac{3^n + (-3)^n}{3^n}$
3. $(\sqrt{n+1} - \sqrt{n})\sqrt{n}$
4. $(n - \sqrt{n^2 - 1}) \log_3 n$
5. $\dfrac{(n+1)!}{(n+1)! - n!}$
6. $\sqrt[n]{\log n}$
7. $\sqrt[3]{n+1} - \sqrt[3]{n}$
8. $\dfrac{(2n)!}{(n!)^2}$
9. $\sqrt[n]{5^n - 4^n}$
10. $2^n - n^n$
11. $\log(1 + \mathrm{e}^n) - \sqrt{n}$
12. $\mathrm{e}^{n-3} - \mathrm{e}^{\sqrt{n^2-4}}$
13. $\dfrac{4^n - 3^n}{4^n + 3^n}$
14. $\dfrac{\sqrt[3]{n}\,\sin(n!)}{n-1}$
15. $\dfrac{(n+1)! - (n-1)!}{(n+2)! - (n+1)!}$
16. $n!(2 + \cos(n!))$
17. $\left(\dfrac{2n^2 - n}{2n^3}\right)^{n^2 \log n}$
18. $\left(1 - \dfrac{3}{\log n}\right)^{2\sqrt{n}}$
19. $(3 + 2n^3)\ \mathrm{tg}\left(\dfrac{4^{-n} + n^3}{n^5 - \sqrt[n]{n^2}}\right)$
20. $\left(1 + \dfrac{n^2+1}{2^n}\right)^{\frac{n!}{2}}$

(3) Tramite la definizione di limite, verificare che per $n \to +\infty$

1. $\log(n^2 + 1) \to +\infty$
2. $\log\left(\dfrac{2}{n^2+1}\right) \to -\infty$
3. $\mathrm{e}^{1/n} \to 1$
4. $\frac{n^2-1}{n+3} \to +\infty$
5. $\frac{1}{n} + \frac{1}{n^2} \to 0$

(4) Determinare estremo superiore, inferiore ed eventuali massimo e minimo dei seguenti insiemi:

1. $A = \{3n - \frac{2}{n};\; n = 1, 2, \dots\}$
2. $A = \{x \in \mathbb{R} : \exists n \in \mathbb{N}$ tale che $n^2x^2 + 2nx + 1 = 0\}$
3. $A = \{x \in \mathbb{R} : \sin(1/x) = 0\}$
4. $A = \{x = n^2 - 4n - 7,\; n = 0, 1, 2, \dots\}$