

## SUMMER SCHOOL PROGRAM

## Course 1 - Applications of human perception in signal processing and analysis

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This course focuses on the intersection of Signal Processing and Human Perception. This is a growing area of interest with many applications ranging from audio and video compression, to speech recognition, virtual reality, and robust system design. Topics to be covered include:

- Human Audio and Visual Perception
- Models of perception
- Signal processing techniques using perceptual error metrics
- Signal processing techniques mimicking human perception

Applications including audio/image/video compression, hearing aids, signal analysis and more...

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## **Course 2 - A Compact Survey of Modern Fourier Analysis**

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Fourier Analysis as found in most mathematical books has little to do with applications of Fourier Analysis as described in engineering or physics books. The course will provide (in 8 units) a fast lane to modern Time-Frequency Analysis (TFA), thus providing mathematical tools which allow the description not only classical questions of Fourier Analysis, but provide versatile tools useful for many applied settings.

The key object of TFA is the so-called Short-Time Fourier Transform (STFT). It provides an isometry from the Hilbert space  $(L^2(\mathbb{R}^d), \|\cdot\|_2)$  into the range of the STFT, which is a closed subspace of  $L^2(\mathbb{R}^{2d})$ , but also a space of continuous functions over  $\mathbb{R}^d \times \widehat{\mathbb{R}}^d$  (phase space). The subspace of all functions with an integrable STFT (known as Feichtinger's algebra  $(S_0(\mathbb{R}^d), \|\cdot\|_{S_0})$ ) is a Banach algebra of continuous, integrable functions on  $\mathbb{R}^d$ , which is also invariant under the (ordinary) Fourier transform (using just Riemann integrals). The dual space  $(S'_0(\mathbb{R}^d), \|\cdot\|_{S'_0})$ , nowadays known as Banach space of mild distributions, can be viewed as a model for all possible *signals*, characterized by the boundedness of the STFT. All the function spaces used in the engineering context (including discrete an continuous, periodic and non-periodic signals) can be viewed as subspaces of  $S'_0(\mathbb{R}^d)$ . Any mild distribution can be approximated by bounded continuous functions and has a natural Fourier transform in  $S'_0(\mathbb{R}^d)$ , which coincides with the classical version in each case, but goes far beyond. Together with the Hilbert space  $L^2(\mathbb{R}^d)$ , these two spaces form the so-called *Banach Gelfand Triple* (BGT)  $(S_0, L^2, S'_0)(\mathbb{R}^d)$ , which behaves in some sense like the triple  $\mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$  of fields, forming a natural chain.

This setting allows the correct mathematical treatment of Dirac measures and Dirac combs. Translation invariant linear systems (TILS) are characterized as convolution operators or alternatively as Fourier multipliers (via some transfer function). Gabor Expansions (atomic series representations of signals as double sums using time-frequency shifted copies of a Gabor atom, such as the Gaussian) allows to characterize the BGT by coefficients in  $(\ell^1, \ell^2, \ell^\infty)$ . Also the foundations of Gabor Analysis can be well described with these tools, including Gabor multipliers or Anti-Wick operators.

In addition there is a kernel theorem, which allows to characterize linear operators by their kernel, but also as a superposition of time-frequency shifts, the so-called spreading representation, which is equivalent to the characterization of time-variant systems (as used for mobile communication) using the Kohn-Nirenberg characterization.

As time permits also questions of discretization and approximation via realizable, constructive methods will be shortly discussed. Most of the terms explained in the course have already been implemented using MATLAB and the speaker has long-standing experience with this tool.

Overall the course will be more like a survey of problems, methods and results offered by this new approach than a discussion of technical details. Some material is found here:

http://www.nuhag.eu/ETH20
https://nuhagphp.univie.ac.at/home/feitalks.php
https://nuhagphp.univie.ac.at/home/feipub\_db.php

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## Course 3 - New Perspectives on Neural Network Design: From Mathematical Foundations to Shape Representation

#### Xue-Cheng Tai<sup>1</sup>

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In this short course, I will present recent research exploring several innovative directions in neural network design, grounded in mathematical modeling and algorithmic insights.

First, we introduce a general framework for constructing neural networks via operator splitting schemes. Starting from a suitable control problem, we discretize it using a carefully designed splitting method. Unrolling this scheme naturally yields new network architectures. We demonstrate this approach with two examples: a simplified UNet and the recently proposed PottsMGNet, both of which emerge naturally from the discretization process.

Second, we offer a new mathematical explanation of the widely used UNet architecture. While UNet has been immensely successful in image segmentation tasks, its underlying structure has lacked rigorous theoretical interpretation. We show that UNet can be viewed as a one-step operator-splitting method for a control problem. Each component of the architecture corresponds to an element in the control formulation, and multigrid techniques are used to decompose the control variables. This perspective not only explains the effectiveness of UNet but also connects it with numerical PDE methods.

Third, we delve into shape representation and segmentation using neural networks, particularly through the lens of the PottsMGNet framework. Encoder-decoder architectures are prevalent in image processing, yet their mathematical foundations remain incomplete. We reinterpret these architectures using the twophase Potts model, formulating the segmentation problem as a control problem in the continuous setting. The problem is then discretized—temporally via operator splitting (yielding PottsMGNet) and spatially via multigrid methods. This leads to a network structure that is provably equivalent to encoder-decoder architectures. PottsMGNet, with a soft-thresholding regularizer, demonstrates robustness to network width, depth, and high noise levels, outperforming or matching state-of-the-art networks in accuracy and Dice score.

We further extend this framework to handle convex shape representation using level set methods. We derive necessary and sufficient conditions for level set functions to represent convex shapes and apply this to variational models for image segmentation. Efficient numerical algorithms are developed and validated through experiments. To improve segmentation in complex images, we incorporate landmark constraints—either enforcing that the boundary passes through specific points or that certain regions belong to foreground or background. These techniques are broadly applicable to convex shape optimization and can be adapted for other applications.

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## **CONFERENCE PROGRAM**

## Tensor Robust CUR for Hyperspectral Data

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Hyperspectral images are often contaminated with noise which degrades the quality of data. Recently, tensor robust principal component analysis (TRPCA) has been utilized to remove noise from hyperspectral images, improving classification accuracy. However, the high dimensionality and size of hyperspectral data present computational challenges both in terms of storage and processing power, especially in the case of TRPCA. The situation is exacerbated when the data is too large to fit in available memory. We present a tensor-robust CUR (TRCUR) algorithm for hyperspectral data compression and denoising. We heavily downsample the input hyperspectral image to form small subtensors; and perform TRPCA on the small subtensors. The desired hyperspectral image is recovered by combining the low-rank solution of the subtensors can be exactly recovered using our proposed TRCUR method. Numerical experiments indicate that our method is up to an order of magnitude faster than performing TRPCA on the original input data while maintaining the classification accuracy.[1]

#### References

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## Tsunami detection for early warning applications: a signal processing perspective.

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Tsunamis represent one of the most potentially destructive natural hazards, to which several coastal areas around the world may be subjected. In recent decades, great effort has been put into building tools and strategies to prevent, or at least alleviate, the possible damage and loss of life. One such tool is a Tsunami Early Warning System (TEWS), i.e. a set of institutions, protocols, common procedures and instrumental measuring networks that is able to reliably alert the population in case a tsunami is approaching a given coastal area. Today, especially after some notable events in the last two decades, such as the tsunamis generated by the 2004 Sumatra earthquake, the 2010 Maule earthquake and 2011 Tohoku earthquake, the development of TEWSs all around the globe has sensibly accelerated.

A fundamental component of TEWSs is direct measurement of the sea level during the propagation of the tsunami. To this aim, the most used instruments are Ocean-Bottom Pressure Gauges (OBPGs) and coastal Tide Gauges (TGs). However, signals recorded by these instruments include components generated by other physical phenomena [1], such as tides, which are the most energetic component, long term oceanographic pressure variation, seismic shaking (for OBPGs) and atmospheric disturbances (for TGs). Thus, realtime signal processing algorithms are needed to extract informations about the travelling tsunami from sea level record.

Many algorithms have been proposed for the realtime detection of tsunami waves. The most used is the so-called Mofjeld's algorithm [2], which is installed in OBPGs of the Deep-ocean Assessment and Reporting of Tsunamis (DART) realtime monitoring system, that are the standard instruments used by the global TEWS managed by NOAA. Mofjeld's algorithm is very efficient in extracting tides from noisy measurements and detecting transient oscillations, such as tsunamis. However, the algorithm is not able to characterise the tsunami, i.e. the residual oscillation does not faithfully represent the tsunami waveform. For this and other reasons, many more tsunami detection algorithms have been proposed, employing a wide variety of signal processing techniques, among which we can find neural networks [3], empirical orthogonal functions [4, 5], slope-based filters [10], harmonic tidal models [6, 7], the Ensemble Empirical Mode Decomposition [8], the Fast Iterative Filtering and IMFogram techniques [9], and many more.

In this talk, an overview of the state of the art of tsunami detection by direct sea level measurement is presented. Particular attention is given to the characteristics that a detection algorithm may benefit from, in terms of tide removal, tsunami waveform extraction and computational efficiency. Issues related to calibration and testing of tsunami detection algorithms are also mentioned. In light of the deployment of OBPGs for operational tsunami forecast for the first time in the Mediterranean [11], particular attention is given to the case where previous data in a given location are scarce or absent.

#### References

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## Machine Learning for Vertical Total Electron Content Forecasting

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Vertical Total Electron Content (vTEC) is one of the most important parameters to study the impact of Space Weather on the ionosphere. vTEC is used to quantify the level of ionization of the upper atmosphere and its variability depends on latitude, time of the day, season and solar cycle phase, which exacerbates especially under severe space weather events. As a contribution to countermeasures for mitigating the impact introduced by the ionospheric threats on communication and navigation systems, several vTEC forecasting models have been developed using different approaches. In our research study, we leverage machine learning techniques to build a multivariate predictive model and design physics-informed loss functions to improve the training process specifically to cope with the associated events of Space Weather.

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# Time-frequency analysis of non-stationary signals: mode separability and joint transforms

#### <u>Vittoria Bruni</u><sup>1</sup>

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Non-stationary signals typically consist of a superposition of amplitude and frequency modulated (AM-FM) components, commonly referred to as modes. Key challenges in their analysis include the separation of individual modes and the accurate estimation of their instantaneous frequencies (IF) [1].

Time-frequency (TF) analysis has long served as a fundamental tool for addressing these tasks. However, its effectiveness diminishes when dealing with signals whose modes violate the separability condition, leading to significant interference and reduced mode discrimination [2, 3].

In this talk, we first address the problem of interference localization in time-frequency representations. We briefly present a machine learning-based method capable of identifying non-separable regions, even when trained on a small dataset [4]. We then introduce recent results on the semi-supervised separation of non-separable modes through the combined use of TF representations and the Radon Transform. The central idea is that the Radon Transform can project interfering modes from the TF domain into a new space where they become more distinguishable [5]. We discuss the theoretical foundations of this approach and outline the conditions under which the joint application of TF and Radon transforms enables effective mode separation.

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## Numerical solutions to instrumental problems: a Magnetic Resonance tale

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Nuclear Magnetic Resonance (NMR) is a spectroscopic technique that observes nuclear spin transitions excited by radiofrequency pulses in presence of an external magnetic field. Its excellent resolution, its intrinsic quantitativeness, and the atomistic view it can give on the investigated system make NMR one of the best analytical tools both in academic and industrial environment, whether for research or for process control. The raw NMR signal consists of an oscillating electric current, that is then digitized and Fourier-transformed in order to give a spectrum. Such a complex technique requires complex instrumentation. The countless components inside the spectrometer are designed to address specific issues on the acquisition side, in order to limit or remove artifacts that might hamper the readability of the spectrum and to increase the quality of the data. However, there are some residuals that cannot be avoided. For this reason, the NMR community recognized the need to apply numerical methods to solve these problems in a post-acquisition stage, thus improving the analysis that will follow. In this presentation, we will focus on the origin of some of these problems, on the effect they have on the data, and on the processing methods that our group proposed to deal with them.

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### Latest development in time-varying wave-shape functions

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Real-world oscillatory signals present some particular characteristics. They are non-stationary, their oscillatory patterns are far from being merely sinusoidal, and they change with time. We will revisit the Adaptive Non-Harmonic Model (ANHM) [1], which compactly encodes non-stationarities and non-sinusoidal oscillatory patterns. We will move then to more versatile models that allow for time-varying waveforms [2]. Then, we will finish discussing the latest developments related to a fully adaptive approach that encodes waveform variability due to non-integer harmonicity and amplitude modulations [3]. We will employ our model for three popular signal processing tasks: denoising, decomposition, and segmentation. We will illustrate with three real-world biomedical examples. The denoising of an electroencephalograph signal will be offered and the results compared to previous methods. For decomposition, our proposal is able to recover the composing waveforms more accurately by considering the time variations from the harmonic amplitude functions when compared to existing methods. Finally, the algorithm is used for the adaptive segmentation of synthetic signals and an electrocardiograph of a patient undergoing ventricular fibrillation.

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## Volcano Monitoring for Aviation Safety improved by real-time plume detection with doppler radar and its application to Etna volcano

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Monitoring the explosive eruptions that produce volcanic plumes contaminating atmosphere is a strategic objective of the volcano observatories selected by international civil aviation authority for enforcing the air traffic safety. The frequent explosive eruptions from Etna summit craters (more than 200 lava fountains occurred in the last 25 years), represent a big problem but also an opportunity to develop new geophysical techniques to detect these phenomena and to measure the key parameters for modelling their dispersal in atmosphere, with the aim of producing an accurate forecast. Monitoring must be done in real-time and under any weather condition, and the radar remote sensing technique is able to achieve this goal. On Etna, a L-band Doppler radar dedicate to this aim was installed in 2010 in a shelter at 2700 m asl and about 3 km afar from the active volcanic vents. To enhance this task in 2015, a S-band Doppler radar was added in the same location. During this year, a sounding X-band Doppler radar and a meteorological X-band Doppler radar will be installed at lower altitude on Etna flanks.

For S-band radar data processing, we developed an automatic chain that has been used to analyze the radar signal recorded during 23 episodes of paroxysmal explosive activity that occurred at the South East Crater during the second half of 2021 [1]. This tool is dedicated to the Monitoring Room of the INGV Osservatorio Etneo (INGV-OE) in Catania. Its work starts by acquiring the Doppler spectra, which are the basic data coming from the radar; each spectrum stores the radar reflectivity factor in decibel (dBZ) generated by the sensed remote objects for each Doppler velocity  $v_D$  (from  $v_{D,min} = -150 \text{ ms}^{-1}$ to  $v_{D,max} = 150 \text{ ms}^{-1}$ ). Then, this data is cleaned, processed and converted into four time series, one for each range bin. These time series contain the dBZ values over time, which are a proxy of the volume of the pyroclastic materials that are reflecting back the radar pulse in that moment. Therefore, the time series rise during an eruptive event, and the range of assumed dBZ values depends on the type of eruptive activity, permitting us to detect the occurrence of strombolian explosive activity and its possible evolution to lava fountains.

To identify the onset of strombolian activity and the subsequent evolution into a lava fountain, empirical thresholds of radar time-series were extracted on the base of the thermal and visible images acquired by the Etna surveillance camera network, to achieve a robust monitoring tool that automatically operates 24/7. This tool improves the capacity to detect volcanic explosive activity, which is strategic for the task of the volcano observatory to inform air traffic controllers through the issuing of the VONA (Volcano Observatory Notice for Aviation) messages on the eruptive phenomena that produce volcanic ash dispersal in the atmosphere, with the aim to avoid the contaminated air space and improving aviation safety.

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## Tracking Explosive Volcanic Activity Through Multiparametric Signal Processing

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Explosive volcanic eruptions produce complex and rapidly evolving signals across different physical domains—thermal, acoustic, visual, and deformation. In this talk, a multiparametric approach to monitoring these phenomena is presented, using custom field setups that record thermal images, high-speed video, acoustic signals and displacement data during explosive activity at Stromboli volcano.

By applying signal processing techniques to these diverse data streams, we can identify patterns and extract quantitative features such as eruption timing, particle velocity, and subtle changes in ground deformation before, during, and after explosions, revealing how eruptions begin, evolve, and end.

One key method involves the use of kymographs—visual representations of how eruption features change over time and space—paired with synchronized acoustic, video and deformation signals to gain a clearer understanding of each event.

This work highlights how signal processing methods can enhance the study of natural phenomena and support real-time monitoring of hazardous events. It also points to open challenges where more advanced tools, such as machine learning or time-frequency analysis, could further improve our ability to interpret complex geophysical signals.

## Explaing the impact of data augmentation with Quantum Harmonic Analysis (QHA) Tools

<u>Monika Dörfler<sup>1</sup></u>

Department of Mathematics University of Vienna Joint work with Henry McNulty and Franz Luef NTNU Trondheim Norway

Data, which we strive to use, interpret, classify, usally undergoes some kind of representation; that is, each data point is written in some dictionary of building blocks by means of coefficients. The building blocks may or may not be derived from the data themselves, they may or may not be physically meaningful and they may or may not depend on few fixed parameters. In any case, we are often interested in using augmentation, which can happen along the dimensions given by the representation coefficients. Augmentation is a well-established and successful tool in many data science applications, however, often heuristic and not well-studied mathematically.

We give new insights on the impact of augmentation, which we interpret as a generalized convolution in a QHA context. Using surprising insights from QHA, we provide a possible explanation of the benefits of augmentation. The basic theory has been published in [1] and [2]

- Monika Dörfler, Franz Luef, Eirik Skrettingland. Local structure and effective dimensionality of time series data sets, Applied and Computational Harmonic Analysis, Volume 73, 2024, 101692, ISSN 1063-5203, https://doi.org/10.1016/j.acha.2024.101692.
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### The Universal Usefulness of The Banach Gelfand Triple

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#### 25.05.2025

We give a summary of the universal usefulness of THE Banach Gelfand triple (BGT)  $(S_0, L_2, S_0^*)(\mathbb{R}^d)$ . Originally developed in the context of time-frequency or Gabor Analysis it has been found to be extremely useful in many other branches of mathematical analysis. It starts from the chain  $S_0 \Rightarrow L_2 \Rightarrow S_0^*$  of Banach spaces. Using local Fourier bases one can identify these spaces with the canonical BGT  $(\ell^1, \ell^2, \ell^\infty)$ . The  $w^*$ topology in  $S_0^*$  describes the natural type of convergence, corresponding to coordinate-wise convergence in  $\ell^\infty$ . Compared to the Schwartz theory of tempered distributions this approach offers various simplifications, while still covering many important aspects of distribution theory relevant applications in engineering or physics.

It appears to be highly plausible to consider the space  $S_0^*$  of mild distributions  $\sigma$  as the universe of all reasonable SIGNALS, while the test functions  $f \in S_0$  take the role of measurements via  $\sigma \mapsto \sigma(f)$ . Obviously the mapping  $(\sigma, f) \to \sigma(f)$  is bilinear and bounded (by definition). In this sense the approach is close to the Dirac setting using the Bra/Ket notation. It also suggests approximation schemes which allows to set up a computational approach to the objects arising in this context.

The corresponding Kernel Theorem allows to identify linear operators T from  $S_0$  to  $S_0^*$  with their (distributional) kernel  $K(x, y) \in S_0^*$ . This opens up the way to various other representations of operators, such as the Wigner transform or the spreading representation.

Among others this opens up a path to basic concepts of Quantum Harmonic Analysis (QHA), Classical Fourier Analysis, System Theory or the classical Shannon Sampling Theorem.

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## ARISTIDE: Artificial Intelligence based Forecasting of Large-Scale Traveling Ionospheric Disturbances over Europe

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Large-Scale Traveling Ionospheric Disturbances (LSTIDs) are perturbations in ionospheric plasma density that can severely impair satellite-based navigation and positioning systems. ARISTIDE is an AI-based early warning system designed to forecast LSTID occurrences throughout Europe. The system exploits geophysical drivers, such as solar wind parameters and geomagnetic indices, as predictive inputs. LSTID events are first identified via image segmentation techniques applied to keograms generated from detrended Total Electron Content (dTEC) data. These events serve as labels for a time series classification task aimed at predicting LSTID occurrences based on prior geophysical conditions. The classification model, QUANT [1], a lightweight interval-based algorithm, was trained and validated on data from 2022 to 2024, achieving an F1 score of 0.61. To enhance interpretability, ARISTIDE incorporates a two-level explainable AI (XAI) framework: Permutation Feature Importance for global model insights and TimeSHAP [2] for local explanations at event level. This approach not only provides timely and reliable LSTID forecasts, but also contributes to a deeper understanding of the ionospheric dynamics driving their formation.

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## Ionospheric features from spectral properties of LOFAR data: The Mother's Day Superstorm

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June 14, 2025

The May 2024 Mother's Day superstorm, classified as a G5-level geomagnetic event, was the strongest since November 2003. While the ionospheric response to such extreme space weather is typically monitored by conventional tools like Global Navigation Satellite System (GNSS), ionosondes, and ground-based magnetometers, this presentation underscores the unique and pivotal contributions that LOFAR can provide to space weather investigation. LOFAR's dense core stations enable detection of fine-scale ionospheric disturbances  $\sim 100$  meters), while its international stations can provide a broader, mid-latitude European perspective. Moreover, the system's dual-antenna design—comprising Low Band (LBA) and High Band (HBA) antennas—supports multi-frequency observations, offering unprecedented resolution in probing ionospheric dynamics.

During the storm's main phase, LOFAR LBA observations revealed pronounced signal fading and absorption, correlating with the dramatic equato-rward expansion of the auroral oval into mid-latitude Europe. This phenomenon, indicative of enhanced ionization and D-layer absorption at lower frequencies, was simultaneously confirmed by Total Electron Content (TEC) maps from GNSS and with observed changes in ionosonde parameters such intense and blanketing E-Region echoes, primarily occurring at virtual heights between approximately 100 and 200 km, alongside substantial deflections in local geomagnetic field components registered by magnetometers. As the storm progressed into its recovery phase, LOFAR HBA measurements captured intense scintillation events, directly attributable to the presence of fast-moving (approaching 800 m/s), small-scale ionospheric irregularities.

To determine these velocities, two independent techniques were employed. The first technique used crosscorrelation of signals from different LOFAR core stations. This method involves analysing the time lag between signals received by pairs of core stations to find the best cross-correlation times of signal power time series, operating under the frozen-in approximation (Taylor's hypothesis). The second technique focused on the determination of Fresnel's frequency from single LOFAR antennas. Additionally, the comparison of these dual techniques allowed the estimation of an effective altitude for these scintillating structures. By iteratively adjusting the assumed altitude of the irregularity layer until both algorithms yielded the same velocity, an effective, time-resolved estimate of the ionospheric altitude was derived. This approach revealed the presence of uplifted ionospheric features extending beyond 1500 km. Such high-altitude phenomena were largely undetectable by conventional high-frequency (HF) ionosondes due to limitations such as severe D-layer absorption, pervasive G-conditions (where the F2 layer is significantly depleted), or simply altitude range constraints. This novel capability of LOFAR provides crucial complementary insights where traditional HF radars may be ineffective.

In conclusion, by meticulously integrating LOFAR data with established space weather datasets, we not only validate LOFAR's efficacy as a powerful instrument for ionospheric research but also highlight its distinctive advantage in resolving previously unobservable storm-time ionospheric features.

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## Automatic Detection of Large-Scale Travelling Ionospheric Disturbances using GNSS Data and Image Processing Techniques

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Large-scale travelling ionospheric disturbances (LSTIDs) are irregularities affecting the ionosphere from auroral to low latitudes, with significant implications for radar systems and satellite navigation. To address the need for rapid detection and forecasting, we present an autonomous detection method based on traveltime diagrams of detrended Total Electron Content (TEC) data over the European region. Our approach utilizes a network of GNSS dual-frequency receivers, selected through a K-Means clustering algorithm to optimize spatial coverage. By detrending the geometry-free linear combination of phase measurements, we generate daily travel-time diagrams that capture the meridional propagation of LSTIDs. We then apply image processing techniques to highlight regions of interest and extract propagating features with both positive and negative amplitudes. These features are fitted using the RANSAC method to estimate latitudinal velocities. Analysis of data spanning 2021–2024 shows that typical LSTIDs in the European sector travel at a speed of  $641 \pm 222$  m/s and have a period of  $61 \pm 14$  minutes. Moreover, correlations with auroral electrojet and geomagnetic indices reveal that increases in auroral currents lead to faster, more pronounced, and shorter-period disturbances, in line with previous studies. This work validates our detection methodology and establishes a foundation for future machine learning-based forecasting experiments of ionospheric disturbances.

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## Non-Gaussian Features in Ionospheric Scintillation

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Gaussian distributions, commonly associated with random processes, are among the most widely used tools in statistical modeling. However, despite their broad applicability, many natural physical systems display behaviors that deviate from Gaussian assumptions. In this context, the presentation explores the non-Gaussian statistical characteristics evident in ionospheric scintillation.

Distortions in Global Navigation Satellite System (GNSS) radio signals, caused by ionospheric irregularities, are examined through the probability distributions of signal amplitude fluctuations. A detailed analysis of these distributions provides insights into the underlying mechanisms responsible for signal distortion. In particular, the use of higher-order statistical moments offers a practical approach to evaluating the appropriateness of Gaussian noise models.

More specifically, skewness and kurtosis serve as key descriptors of the statistical distributions associated with ionospheric structures across both large and small spatial scales. Empirical data from the polar regions indicate that amplitude scintillation distributions are predominantly leptokurtic, exhibiting little to no skewness over a wide range of spatial and temporal scales. Furthermore, the propagation of radio signals through the ionosphere—acting as a scattering medium—produces long-range correlation patterns that further reveal the presence of non-Gaussian features.

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## Inverse problem theory and model parameters

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Inverse problem theory encompasses a diverse range of techniques applicable to a broad spectrum of data fitting challenges across various disciplines — not only geophysics, but also engineering, economics, genetics, and many others. Techniques can generally be collectively well understood within the framework of Bayesian inference. In this concise overview, I aim to provide a summary of some of the available techniques, accompanied by specific application insights and advantages. I will draw upon examples primarily from my own experience in seismic tomography, but the methods are highly generalizable and can be applied to any data fitting endeavor.

As shown by Tarantola [1], Bayesian inference provides a viable general framework for structuring the general solution to the inverse problem, where we seek complete information on model parameters m that best reproduce observations d through some theoretical relation allowing to compute d given m, as g(m) = d. The most general answer, representing the full complexity of the solution, can be found using a full Monte Carlo approach, but in the most common cases this is not feasible because computationally prohibitive. Two different paths may then open. On one side, one can tweak the Monte Carlo process and make it viable by using many sequential lower-dimensional model space projections along what are known as *reversible-jump* Markov chains. Following another direction, one can — more efficiently — seek and characterize a 'best' solution, for instance adding an additional constraint and executing some multi-objective optimization. Interesting alternative approaches are based on biological analogies of natural success, to implement metaheuristic processes, that may be highly effective although lacking a true theoretical justification.

In this concise overview, I will emphasize intuition over mathematics. I warmly encourage listeners to explore in more rigorous depth, if they wish, the topics that better resonate with their specific applications.

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# Fractional Derivative Variational Model for Additive Signal Decomposition

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We present a novel variational model for the additive decomposition of 1D noisy signals. The model relies on sparsifying fractional-order derivatives of the sought-for components to capture intricate signal structures. To efficiently solve the resulting optimization problem, an alternating direction method of multipliers-based algorithm is developed. Furthermore, a bilevel optimization framework is proposed to automatically select "optimal" values of all the free parameters in the model, including the orders of the sparsified fractional derivatives. Preliminary results validate the effectiveness of the proposed approach in accurately decomposing noisy signals, even in the presence of abrupt changes.

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### Space weather predictions: state of the art and challenges

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Solar Coronal Mass Ejections (CMEs) are large-scale eruptive events in which large amounts of plasma (up to 10<sup>13</sup>-10<sup>16</sup> g) and magnetic field are expelled into interplanetary space at very high velocities (typically 450 km/s, but up to 3000 km/s). When sampled in situ by a spacecraft in the interplanetary medium, they are termed Interplanetary CMEs (ICMEs). They are the major drivers of "space weather" and the associated geomagnetic activity. The detectable effects of space weather on Earth appear in a broad spectrum of time and length scales and have various harmful effects on human health and the technologies on which we are ever more dependent. Severe conditions in space can hinder or damage satellite operations, and communication and navigation systems. They can even cause power grid outages, leading to various socio-economic losses. To mitigate these effects or, at least, lower their impact, numerical physics-based models are developed to unravel the physics behind these phenomena and predict the effects a few days in advance.

To predict the impact of a CME and its so-called geo-effectiveness, it is essential to consider the internal magnetic structure of the CMEs, as the sign of the magnetic field component perpendicular to the equatorial plane upon arrival at Earth is a key parameter. I will first present the challenges in modelling the solar wind and the evolution of CMEs. Then, I will go deeper into the latest results on magnetic flux-rope models for CMEs implemented in EUHFORIA, our heliospheric wind and CME evolution model (Pomoell & Poedts, 2018). These state-of-the-art models include the spheromak model, Fri3D (Flux-Rope in 3D), the radially contracted spheromak, two analytical and self-similar toroidal CME models (based on the Soloviev and Miller-Turner solutions), and a novel 'horseshoe' CME model (Maharana et al. 2024). I will also mention our novel modified Titov-Démoulin and RBSL flux-rope CME models that we can launch from the low solar corona via our global MHD coronal model COCONUT (Perri, Leitner et al. 2022). These CME models are integrated into Sun-to-Earth model chains. They are used to predict the geo-effectiveness of CME impacts on Earth, quantified by magnetosphere models and models for Dst, Kp, etc. indices.

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## Transformers in Time Series Forecast

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The introduction of the first Attention Mechanisms in 2017 [1] opened new possibilities for modeling sequence data, which spread from Natural Language Processing to other domains, like Computer Vision, Audio Modeling and Time Series Forecasting. The resulting Transformer Architecture showed promising improvements in such fields, especially because of its improved vector context representation built through the attention layer and its extended processing window [2]. Although it's mainly applied in Natural Language Processing through the recently emerged Large Language Models (like GPT, Gemini, etc.), several research teams are experimenting with such a technology in other domains, like Time Series Forecasting [3] [4]. The Transformer architectures allows to capture long-range time step dependencies, which resulted in better performance with respect to other methods for both univariate and multivariate time series forecasting [2].

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- [3] Haixu Wu Et al. "Autoformer: Decomposition Transformers with Auto-Correlation for Long-Term Series Forecasting" January 2022
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## Detection of electromagnetic anomalies emerging from the ionospheric background

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For over two decades, the scientific literature has documented ionospheric disturbances, including electromagnetic emissions, linked to strong earthquakes. Yet, accurate identification of these disturbances critically depends on establishing a clear ionospheric background in the absence of seismic activity and other potential input (e.g., transient change in solar activity).

Employing the Fast Iterative Filtering (FIF) technique, a new methodology for estimating the electromagnetic background within the ionosphere has been developed. This approach involves a multiscale statistical analysis that leverages electric and magnetic field data from the China Seismo-Electromagnetic Satellite (CSES-01). The methodology incorporates the geomagnetic conditions and the diurnal-nocturnal differences within the ionosphere. This enables the generation of distinct background maps that inherently consider the ionosphere's intrinsic variability.

For robust analysis, only EM signals exhibiting statistical significance relative to the defined background should be considered as legitimate events for further investigation. Comparisons between observations in the vicinity of seismic activity and the corresponding background have revealed the presence of notable events in both co-seismic and pre-seismic phases, across several regions characterized by elevated seismic activity.

Furthermore, a statistically rigorous definition of anomalies is introduced, to discriminate between rare and anomalous events.

## Latent Mode Decomposition

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We introduce Variational Latent Mode Decomposition (VLMD), a new algorithm for extracting oscillatory modes and associated connectivity structures from multivariate signals. VLMD addresses key limitations of existing Multivariate Mode Decomposition (MMD) techniques -including high computational cost, sensitivity to parameter choices, and weak modeling of interchannel dependencies. Its improved performance is driven by a novel underlying model, Latent Mode Decomposition (LMD), which blends sparse coding and mode decomposition to represent multichannel signals as sparse linear combinations of shared latent components composed of AM-FM oscillatory modes. This formulation enables VLMD to operate in a lower-dimensional latent space, enhancing robustness to noise, scalability, and interpretability. The algorithm solves a constrained variational optimization problem that jointly enforces reconstruction fidelity, sparsity, and frequency regularization. Experiments on synthetic and real-world datasets demonstrate that VLMD outperforms state-of-the-art MMD methods in accuracy, efficiency, and interpretability of extracted structures. The arXiv version of the full article is available here [1].

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## Spherical Iterative Filtering For Icy Moon Exploration: The Study Case Of Ganymede

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The gravitational field of a planetary body directly reflects the distribution of mass within its interior. Accurately identifying the contributions from individual internal layers is essential for understanding the planet's internal structure. This is especially important for icy moons, whose interiors are believed to consist of a dense, rocky core beneath a hydrosphere made up of a subsurface ocean and outer ice shells. In such layered systems, large-scale gravity anomalies are generally associated with the deeper rocky interior, while smaller-scale variations often originate from the hydrosphere or icy crust. By expressing the gravitational potential in terms of spherical harmonics [1], the field can be broken down into spatial frequency components, enabling the use of signal processing methods. However, due to their intrinsic properties, conventional decomposition methods, such as the Fourier or wavelet transforms, struggle to handle non-stationary, nonlinear signals on spherical surfaces.

Here, we present the application of a new mathematical technique called "Spherical Iterative Filtering" (SIF), tailored for analysing non-stationary signals defined on spheres, for the gravitational data analysis. SIF extends the established Iterative Filtering (IF) algorithm, originally created for one-dimensional time series analysis [2], to spherical geometries. IF has proven effective across multiple fields [3], and its speed and robustness have been significantly improved through the Fast Iterative Filtering (FIF) version, which guarantees convergence and leverages the Fast Fourier Transform for acceleration [4]. SIF adapts this strategy to the spherical domain, extracting oscillatory components named "Intrinsic Mode Surfaces" (IMSs). Unlike traditional approaches, SIF does not rely on predefined bases or assumptions, allowing it to adaptively extract signal components while maintaining their non-stationary nature.

As a demonstration, we apply SIF to simulated gravity data for Ganymede, the largest moon of Jupiter and

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a key target of the European Space Agency's upcoming JUICE mission. This mission is expected to deliver high-resolution gravity measurements critical for investigating Ganymede's interior. Based on a reasonable structural model of Ganymede [6], we use SIF to break down its synthetic gravity field, successfully isolating signals from the rocky core and the surrounding hydrosphere. Notably, this separation is achieved without any external guidance or prior assumptions about the internal layering. While these findings are based on synthetic data and subject to model uncertainties, they serve as proof of concept. The results from SIF can offer a valuable preliminary tool for narrowing down parameter spaces prior to applying more computationally intensive inverse modelling techniques, thereby supporting gravity data interpretation workflows.

In conclusion, Spherical Iterative Filtering presents a robust and adaptable method for analysing planetary gravitational signals, especially in cases involving complex, stratified interiors like those of icy moons. Its ability to break down spherical, non-stationary signals in a data-driven, assumption-light manner positions it as a promising approach for future geophysical studies and planetary exploration.

Acknowledgements: E.S.M. and G.M. acknowledge support from the Italian Space Agency (project 2023-6-HH.0). This research has been conducted within the framework of the Italian national inter-university PhD programme in Space Science and Technology.

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## Critical points of the anisotropic isoperimetric problem

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A critical point of the anisotropic isoperimetric problem is a (Lipschitz) domain such that the first variation of the anisotropic boundary area is zero for all volume preserving deformations. After introducing the problem, I review some known results and a few important open questions.

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## A Mathematical Explanation Of Unet

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For problems in image processing and many other fields, a large class of effective neural networks has encoder-decoder-based architectures. Although these networks have made impressive performances, mathematical explanations of their architectures are still underdeveloped. In this paper, we study the encoderdecoder-based network architecture from the algorithmic perspective and provide a mathematical explanation. We use the two-phase Potts model for image segmentation as an example for our explanations. We associate the segmentation problem with a control problem in the continuous setting. Then, multigrid method and operator splitting scheme, the PottsMGNet, are used to discretize the continuous control model. We show that the resulting discrete PottsMGNet is equivalent to an encoder-decoder-based network. With minor modifications, it is shown that a number of the popular encoder- decoder-based neural networks are just instances of the proposed PottsMGNet. By incorporating the Soft-Threshold-Dynamics into the PottsMGNet as a regularizer, the PottsMGNet has shown to be robust with the network parameters such as network width and depth and achieved remarkable performance on datasets with very large noise. In nearly all our experiments, the new network always performs better or as good on accuracy and dice score than existing networks for image segmentation.

This talk is based on joint works with Raymond Chan, Hao Liu and Lingfeng Li, [1, 2]

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## Conformal Prediction: Uncertainty Quantification to Humanise Models

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Quantifying uncertainties of Machine Learning models is crucial to improve their reliability, accurately assess risks, and make more robust decisions. By quantifying and understanding uncertainty, we can build more reliable and trustworthy systems. Imagine that we have a model that predicts whether or not a CT scan contains a tumour: traditional approaches tend to provide binary predictions, while not providing information on the confidence of the model in each prediction.

Conformal Prediction (CP) is a framework for uncertainty quantification that offers an estimate of confidence in the model predictions: instead of providing just a point estimate, it provides a set of possible outcomes (prediction set), together with a measure of confidence in each outcome. These prediction sets come with a mathematical guarantee of coverage of the true outcome, ensuring that they will detect at least a pre-fixed percentage of true values. CP is a model-agnostic paradigm, which requires no retraining of the model and makes no major assumptions about the distribution of the data. [1, 2]

Humans, when faced with uncertainty, tend to express indecision and offer alternatives [3]. We will see that CP can be a key tool to include a human in the decision-making loop, once the 'humanised' machine is able to express its uncertainty. CP therefore offers a robust framework that allows stakeholders to make more informed decisions, even more so in high-risk sectors such as healthcare, finance, and autonomous systems.

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## Deep Learning in Non-Stationary Signal Decomposition

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The decomposition of non-stationary signals is a fundamental yet complex challenge in time-frequency analysis. While traditional methods such as Empirical Mode Decomposition have laid the foundation for signal decomposition, they suffer from limitations including boundary effects, mode mixing, and low robustness to noise. To address these challenges, recent advances have turned to deep learning approaches, culminating in the development of the Iterative Residual Convolutional Neural Network (IRCNN)[?]. IRCNN leverages convolutional neural networks, residual structures, and nonlinear activation functions to perform decomposition by learning the local average of signals in an end-to-end manner. Building upon this, an enhanced framework, IRCNN+, integrates advanced deep learning components—such as multi-scale convolution, attention mechanisms, and total variation denoising—to improve feature representation, smoothness, and adaptability. Comprehensive evaluations demonstrate that both IRCNN and IRCNN<sup>+</sup> outperform conventional methods in terms of decomposition accuracy, robustness to noise, and handling of mode mixing and boundary effects, offering a promising direction for scalable and effective non-stationary signal decomposition.

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