Master Course in Mathematical Engineering — 2020/21
Advanced Analysis 1 – 6 CFU
Lecturer: C. Lattanzio

Distributions. Locally integrable functions. The space of test function $\mathcal{D}(\Omega)$. Distributions. Distributions associated to locally integrable functions. Singular distributions. Examples. Operations on distributions: sum, products times functions, change of variables, restrictions, tensor product. Differentiation and his properties; comparison with classical derivatives. Differentiation of jump functions. Partition of unity. Support of a distribution; compactly supported distributions.

Convolution. Convolution in L^p spaces. Regularity of the convolution. Regularizing sequences and smoothing by means of convolutions. Convolution between distributions and regularization of distributions. Denseness of $\mathcal{D}(\Omega)$ in $\mathcal{D}'(\Omega)$.

Sobolev spaces. Definition of weak derivatives and his motivation. Sobolev spaces $W^{k,p}(\Omega)$ and their properties. Interior and global approximation by smooth functions. Extensions. Traces. Embeddings theorems: Gagliardo–Nirenberg–Sobolev inequality and embedding theorem for $p < n$. Hölder spaces. Morrey inequality. Embedding theorem for $p > n$. Sobolev inequalities in the general case. Compact embeddings: Rellich–Kondrachov theorem, Poincaré inequalities. Embedding theorem for $p = n$. Characterization of the dual space H^{-1}.

Second order parabolic equations. Definition of parabolic operator. Weak solutions for linear parabolic equations. existence of weak solutions: Galerkin approximation, construction of approximating solutions, energy estimates, existence and uniqueness of solutions.

First order nonlinear hyperbolic equations. Scalar conservation laws: derivation, examples. Weak solutions, Rankine-Hugoniot conditions, entropy conditions. L^1 stability, uniqueness and comparison for weak entropy solutions. Convergence of the vanishing viscosity and existence of the weak, entropy solution. Riemann problem.

Textbooks:

- H. Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations.* Universitext, Springer.
- C.M. Dafermos, *Hyperbolic Conservation Laws in Continuum Physics*, Springer.
- L.C. Evans, *Partial Differential Equations.* Graduate Studies in Mathematics, Vol. 19, AMS.
- G. Gilardi, *Analisi 3.* McGraw–Hill.
- V.S. Vladimirov, *Equations of Mathematical Physics.* Marcel Dekker, Inc.