PhD COURSES 2025/26

Titles

1. **Exact Response Theory for nonequilibrium dynamics**, 10h, M. Colangeli
2. **Polynomial identities in algebras: a combinatorial approach**, 15h, Elena Pascucci (Sapienza)
3. **Selected topics in geometry**, 10h, B. Nelli, G. Pipoli
4. **Perturbation Methods for the Stability Analysis of Dynamical Systems**, 10h, M. Ferretti, Daniele Zulli
5. **Mathematical models for economic equilibria**, 10h, M. Giuli
6. **Selected topics in probability: an introduction to Rough Paths**, 10h, Lucio Galeati
7. **Selected topics in numerical analysis**, 10h, Stefano Di Giovacchino
8. **Towards Metamaterials: Variational methods in continuum mechanics**, 10h, Alessandro Ciallella, Francesco Dell'Isola
9. **Random dynamical systems**, 10h, Renato Colucci (Università Politecnica delle Marche)
10. **Besov spaces and application to linear transport equations**, 10h, Gennaro Ciampa, S. Spirito
11. **Optimal Control and applications**, 10h, M. Palladino, Franco Rampazzo (Padova)
12. **Introduction to Quantum Computing**, 14h, L. Guidoni, Federico Mattei (IBM Italia), Dibyajyoti Chakravarti
13. **Mathematics for Signal processing**, 15h, A. Cicone
14. **Measure-valued solutions to nonlinear PDEs**, 4h, Agnieszka Swierczewska-Gwiazda (University of Warsaw)

An additional course (6 hours) by Jeffrey Rauch (Univ. Michigan) will be held on April 8-10 2026.

Abstracts

1) **Exact Response Theory for nonequilibrium dynamics**, 10 hours, Matteo Colangeli

The study of particle systems out of equilibrium is part of a large research endeavor which aims at unraveling the dynamical foundations of statistical mechanics. This research line witnessed a substantial advancement, in the last few decades, with the discovery of the Fluctuation Relations and the systematization of response theory for dissipative dynamics. Prompted by recent findings in the field of nonequilibrium molecular dynamics, we review the notions of the dissipation function and T-mixing for non-invariant measures. We also provide a dynamical-systems interpretation for the dissipation function and shed light on some of the prominent aspects of Clausius's legacy, e.g. the second law of Thermodynamics and the onset of the time's arrow.
In this series of lectures the analysis of deterministic models will be pursued in detail and the extension to stochastic dynamics will also be discussed.

2) **Polynomial identities in algebras: a combinatorial approach**, 15 hours, Elena Pascucci (Sapienza - Roma)

The course is an introduction to the theory of polynomial identities for associative algebras and aims to show how tools from representation theory can be applied to study algebras with polynomial identities. In the first part, we will review fundamental concepts from the representation theory of finite groups, focusing in greater detail on the case of the symmetric group. Next, we will delve into PI theory. We will focus on the concepts of polynomial identities, T-ideals, and PI algebras, and explore how these can be studied using numerical invariants related to their identities. Finally, we will emphasize how the combinatorial tools discussed earlier can be used to investigate the T-ideal of identities satisfied by a given algebra.

3) **Selected topics in geometry**, 10 hours, Barbara Nelli, Giuseppe Pipoli

The course aims to present a selection of advanced geometry topics; the choice of topics covered as well as the reference teacher varies from year to year in order to have a very wide spectrum of offerings with modern and frontier topics. The choice of topics can also be made based on the students who will follow the course.

4) **Perturbation Methods for the Stability Analysis of Dynamical Systems**, 10 hours, Manuel Ferretti, Daniele Zulli

The course introduces the basics of the perturbation analysis for weakly nonlinear dynamical systems, with special reference to the multiple scale method for ordinary differential systems. The following topics are addressed: eigenvalue and eigenvector sensitivity analysis; initial value problems: straightforward expansions; the multiple scale method: basic aspects and advanced topics; Duffing oscillator under external excitation: primary, super-harmonic and sub-harmonic resonances; Duffing oscillator under parametric excitation; multi-d.o.f. quasi-Hamiltonian systems under external/parametric/internal resonances.

5) **Mathematical models for economic equilibria**, 10 hours, Massimiliano Giuli

In science the term *equilibrium* has been widely used in physics, chemistry, biology, engineering and economics, among others, within different frameworks. It generally refers to conditions or states of a system in which all competing influences are balanced. For instance, the economic equilibrium which studies the dynamics of supply, demand, and prices in an economy within several markets, can be modeled as a variational inequality problem. In non-cooperative game involving two or more players, Nash proposed an equilibrium solution in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy. This problem can be reformulated as a fixed point problem. These mathematical models share an underlying common structure that allows to conveniently formulate them in a unique format of equilibrium.

The course is devoted to describe this format and it focuses on the main mathematical tools which are crucial for studying the existence and the stability of the solutions.

6) **Selected topics in probability: An introduction to Rough Paths**, 10 hours, Lucio Galeati

The theory of Rough paths, introduced by Lyons in the '90s, provides a general analytical framework for solving differential equations driven by irregular signals (possibly random, like Brownian motion). Lyons' perspective is to split the problem in two steps, one (purely probabilistic) related to the construction of an "enhanced" signal, and another (purely analytic) concerning solvability of the equation for any realization of such "enhancement". Rough paths allow to bypass the limitations of Itô calculus (lack of continuity of the Itô map, need for a martingale structure) and provide natural explanations for classical stochastic analysis results, like the Wong-Zakai principle.

This course serves as a self-contained introduction to the topic; we will mainly focus on sewing techniques and solvability of differential equations driven by either Young or level-two rough paths. A stochastic analysis background is encouraged but not needed.

7) **Selected topics in numerical analysis**, 10 hours, Stefano Di Giovacchino

The course aims to present a selection of advanced topics in numerical analysis; the choice of topics covered and the reference teacher varies from year to year to have a vast spectrum of offerings with modern and frontier topics. The choice of topics can also be made based on the students who will attend the course.

8) **Towards Metamaterials: Variational methods in continuum mechanics**, 10 hours, Alessandro Ciallella, Francesco Dell'Isola

1. Principle of Virtual Work as a fundamental postulate for mechanics Second Gradient Continuum Mechanics. Hamilton Rayleigh Principle for dissipative systems
2. Generalisation of the concept of Deformation and Stress: Necessary strong form for Equilibrium Conditions Essential and Natural Boundary Conditions
3. Piola Transformations and contact interactions for Second Gradient Continua
4. Edge and Surface contact interactions in second gradient continua: forces and double forces. Representation of contact interactions in terms of stresses, double stresses and shape of Cauchy cuts Limitations of so called Cauchy postulate.
5. Some remarks on relevant aspects of history of mechanics and in particular on the development of the concepts of force, stress and couples.

9) **Random dynamical systems**, 10 hours, Renato Colucci (Università Politecnica delle Marche)

The course represents an introduction to the theory of non-autonomous and stochastic dynamical systems . In the first part (2 hours) we wil resume the basic fact of the theory of autonomous dynamical systems linear operator, absorbing sets, global attractors) and then we pass to introduce non-autonomous dynamical systems (2 hours) with particular emphasis on the study of the asymptotic behaviour of the analysed systems (pullback attraction, pullback attractors). Then, we pass to introduce the theory of random dynamical systems and random attractors (3 hours) for stochastic differential equations (additive an multiplicative noise, cocycles, random absorbing sets). The last part of the course (3 hours) is dedicated to the recent trends in this field suc as weak mean random attractor theory, coloured noise and Wong-Zakai approximations.

10) **Besov spaces and application to linear transport equations**, 10 hours, Gennaro Ciampa - Stefano Spirito.

The course aims at introducing some important tools from Harmonic Analysis, such as the Littlewood-Paley decomposition and the theory of Besov spaces, which are nowadays the building blocks of powerful methods in the analysis of partial differential equations. These tools will then be exploited to prove end-point regularity estimates for linear transport equations with irregular vector fields. The regularity classes of vector fields considered in the present course play an important role in several fluid dynamics models.

11) **Optimal Control and applications**, 10 hours, Michele Palladino, Franco Rampazzo (Padova).

In this course, we will describe basic properties of optimal control problems. We will mainly focus on finite dimensional control problems, with fixed or free end point. In particular, we will discuss the following topics:
1. Controllability and existence of optimal controls;
2. Necessary optimality conditions (the Pontryagin Maximum Principle)
3. Higher-order necessary
4. Dynamic Programming Principle, Hamilton-Jacobi Equations and Viscosity solutions.
5. Applications to Machine Learning and Artificial Intelligence.

12) **Introduction to Quantum Computing**, 14h

Speakers: Leonardo Guidoni (UnivAQ); Hands-on tutorial lead by Experts from IBM-Italia and UnivAQ (Federico Mattei - IBM Italia, Dibyajyoti Chakravarti UnivAQ)

Structure of the course: 8 hours of lectures + 6 hours of computer lab hands-on tutorial

The present short course is a joint PhD course between the PhD in Mathematics and Models, the PhD in Physics and Chemistry and the PhD in Information and Communication Technology.

The aim of the course is to provide students with different background basic knowledge of quantum computation, both from the theoretical and the computational side. The course will consist in theoretical lectures as well as hands-on tutorial with practical exercise on Python simulators and real quantum devices in the IBM-Q cloud.
Topics: General overview on quantum computation. Introduction to Quantum Mechanics and Qubits. Quantum circuits and algorithms. Single and double Qubit gates with examples. Present and future applications. Perspective of quantum computation and practical implementation of algorithms on the IBM-Q quantum computer and simulator.

13) **Mathematics for Signal processing**, 15 hours, Antonio Cicone

One classical problem in many applied fields of research, like Geophysics, Medicine, Engineering, Economy and Finance, is, given a signal, how to extract hidden information and features contained in it, like, for instance, quasiperiodicities and trends.
Standard methods like Fourier and Wavelet Transform, historically used in signal processing, proved to be limited when nonlinear and nonstationary phenomena are present. For this reason in the last two decades several new nonlinear methods have been developed by many research groups around the world and they have been used extensively in the applications.
In this course we will review the Empirical Mode Decomposition method and derived techniques, and introduce the Iterative Filtering technique and its generalizations. We will discuss their theoretical and numerical properties, show their limitations and discuss open problems.
During the course some applications will be presented as well as an introduction to the Matlab codes available online[1].

Lessons Plan:

- Description of the course (1/2 h)
- Empirical Mode Decomposition method
  - Review of the algorithm (1 h)
  - Alternative approaches (Ensemble EMD) (1 h)
- Iterative Filtering
  - Review of the algorithm (1 h)
  - Convergence properties in the continuous and discrete setting (1 h)
  - Pitfalls and caveats
    - Boundary Effects (1 h)
    - Jumps and spikes (1 h)
  - Acceleration of the algorithm (Fast Iterative Filtering and Direct Iterative Filtering) (1h)
  - Multidimensional and Multivariate extensions (1 h)
  - Applications (1 h)
- Beyond Iterative Filtering
  - Adaptive Local Iterative Filtering
    - Review of the algorithm (1/2 h)
    - Numerical Analysis (1 h)
  - Resampled Iterative Filtering (1 h)
- Time frequency analysis - IMFogram (1 h)

[1] www.cicone.com

- Instantaneous Frequency and Phase - JADE (1 h)

Final Exam: It will consists of either a short presentation on selected papers or a technical report on numerical experiments.

14) **Measure-valued solutions to nonlinear PDEs**, 4h, Agnieszka Swierczewska-Gwiazda (University of Warsaw)

Solving nonlinear partial differential equations within any reasonable class of generalised solutions is undoubtedly one of the foremost tasks of mathematical analysis, notably when the problems one considers are motivated by physically observable phenomena. This task is considered extremely difficult, and rigorous mathematical treatment of many systems of enormous practical importance (e.g. the fundamental equations of fluid mechanics or conservation laws) remains vastly incomplete.

To make any progress, one repeatedly needs to deal with somewhat feeble notions of solutions formulated as rather abstract objects. Among these notions, the concept of measure-valued solutions stands out as particularly useful and convenient. I will introduce this concept using the simple example of scalar conservation laws and discuss why many of its appealing features fail to carry over to systems of equations. The main part of the lectures will be devoted to the relative entropy method. I will highlight its various advantages and implications, including the weak–strong uniqueness principle, applications to singular limit problems, and insights into long-time stability.