PhD COURSES 2025/26

List of the courses

1. **Exact Response Theory for nonequilibrium dynamics**
2. **Polynomial identities in algebras: a combinatorial approach**
3. **Selected topics in geometry**
4. **Perturbation Methods for the Stability Analysis of Dynamical Systems**
5. **Mathematical models for economic equilibria**
6. **Selected topics in probability**
7. **Selected topics in numerical analysis**
8. **Towards Metamaterials: Variational methods in continuum mechanics**
9. **Random dynamical systems**
10. **Besov spaces and application to linear transport equations**
11. **Optimal Control and applications**

Abstracts

1) **Exact Response Theory for nonequilibrium dynamics**, 10 hours, M. Colangeli.

The study of particle systems out of equilibrium is part of a large research endeavor which aims at unraveling the dynamical foundations of statistical mechanics. This research line witnessed a substantial advancement, in the last few decades, with the discovery of the Fluctuation Relations and the systematization of response theory for dissipative dynamics. Prompted by recent findings in the field of nonequilibrium molecular dynamics, we review the notions of the dissipation function and T-mixing for non-invariant measures. We also provide a dynamical-systems interpretation for the dissipation function and shed light on some of the prominent aspects of Clausius's legacy, e.g. the second law of Thermodynamics and the onset of the time,Äôs arrow.
In this series of lectures the analysis of deterministic models will be pursued in detail and the extension to stochastic dynamics will also be discussed.

2) **Polynomial identities in algebras: a combinatorial approach**, 15 hours, Elena Pascucci, Università di Roma La Sapienza.

The course is an introduction to the theory of polynomial identities for associative algebras and aims to show how tools from representation theory can be applied to study algebras with polynomial identities. In the first part, we will review fundamental concepts from the representation theory of finite groups, focusing in greater detail on the case of the symmetric group. Next, we will delve into PI theory. We will focus on the concepts of polynomial identities, T-ideals, and PI algebras, and explore how these can be studied using numerical invariants related to their identities. Finally, we will emphasize how the combinatorial tools discussed earlier can be used to investigate the T-ideal of identities satisfied by a given algebra.

3) **Selected topics in geometry**, 10 hours, teacher to be assigned.

The course aims to present a selection of advanced geometry topics; the choice of topics covered as well as the reference teacher varies from year to year in order to have a very wide spectrum of offerings with modern and frontier topics. The choice of topics can also be made based on the students who will follow the course.

4) **Perturbation Methods for the Stability Analysis of Dynamical Systems**, 10 hours, M. Ferretti, D. Zulli

The course introduces the basics of the perturbation analysis for weakly nonlinear dynamical systems, with special reference to the multiple scale method for ordinary differential systems. The following topics are addressed: eigenvalue and eigenvector sensitivity analysis; initial value problems: straightforward expansions; the multiple scale method: basic aspects and advanced topics; Duffing oscillator under external excitation: primary, super-harmonic and sub-harmonic resonances; Duffing oscillator under parametric excitation; multi-d.o.f. quasi-Hamiltonian systems under external/parametric/internal resonances.

5) **Mathematical models for economic equilibria**, 10 hours, M. Giuli.

In science the term *equilibrium* has been widely used in physics, chemistry, biology, engineering and economics, among others, within different frameworks. It generally refers to conditions or states of a system in which all competing influences are balanced. For instance, the economic equilibrium which studies the dynamics of supply, demand, and prices in an economy within several markets, can be modeled as a variational inequality problem. In non-cooperative game involving two or more players, Nash proposed an equilibrium solution in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy. This problem can be reformulated as a fixed point problem. These mathematical models share an underlying common structure that allows to conveniently formulate them in a unique format of equilibrium.
The course is devoted to describe this format and it focuses on the main mathematical tools which are crucial for studying the existence and the stability of the solutions

6) **Selected topics in probability**, 10 hours, teacher to be assigned.

The course aims to present a selection of advanced topics in probability; the choice of topics covered as well as the reference teacher varies from year to year in order to have a very wide spectrum of offerings with modern and frontier topics. The choice of topics can also be made based on the students who will attend the course.

7) **Selected topics in numerical analysis**, 10 hours, teacher to be assigned.

The course aims to present a selection of advanced topics in numerical analysis; the choice of topics covered and the reference teacher varies from year to year to have a vast spectrum of offerings with modern and frontier topics. The choice of topics can also be made based on the students who will attend the course.

8) **Towards Metamaterials: Variational methods in continuum mechanics**, 10 hours, A. Ciallella, F. Dell'Isola

Principle of Virtual Work as a fundamental postulate for mechanics Second Gradient Continuum Mechanics. Hamilton Rayleigh Principle for dissipative systems 2. Generalisation of the concept of Deformation and Stress: Necessary strong form for Equilibrium Conditions Essential and Natural Boundary Conditions 3. Piola Transformations and contact interactions for Second Gradient Continua 4. Edge and Surface contact interactions in second gradient continua: forces and double forces. Representation of contact interactions in terms of stresses, double stresses and shape of Cauchy cuts Limitations of so called Cauchy postulate 5. Some remarks on relevant aspects of history of mechanics and in particular on the development of the concepts of force, stress and couples.

9) **Random dynamical systems**, 10 hours, Renato Colucci (Università Politecnica delle Marche)

The course represents an introduction to the theory of non-autonomous and stochastic dynamical systems . In the first part (2 hours) we wil resume the basic fact of the theory of autonomous dynamical systems linear operator, absorbing sets, global attractors) and then we pass to introduce non-autonomous dynamical systems (2 hours) with particular emphasis on the study of the asymptotic behaviour of the analysed systems (pullback attraction, pullback attractors). Then, we pass to introduce the theory of random dynamical systems and random attractors (3 hours) for stochastic differential equations (additive an multiplicative noise, cocycles, random absorbing sets). The last part of the course (3 hours) is dedicated to the recent trends in this field suc as weak mean random attractor theory, coloured noise and Wong-Zakai approximations.

10) **Besov spaces and application to linear transport equations**, 10 hours, G. Ciampa and S. Spirito.

The course aims at introducing some important tools from Harmonic Analysis, such as the Littlewood-Paley decomposition and the theory of Besov spaces, which are nowadays the building blocks of powerful methods in the analysis of partial differential equations. These tools will then be exploited to prove end-point regularity estimates for linear transport equations with irregular vector fields. The regularity classes of vector fields considered in the present course play an important role in several fluid dynamics models.

11) **Optimal Control and applications**, 10 hours, M. Palladino, C. Pignotti, Franco Rampazzo (Università di Padova).

In this course, we will describe basic properties of optimal control problems. We will mainly focus on finite dimensional control problems, with fixed or free end point. In particular, we will discuss the following topics:

1. Controllability and existence of optimal controls;
2. Necessary optimality conditions (the Pontryagin Maximum Principle)
3. Higher-order necessary
4. Dynamic Programming Principle, Hamilton-Jacobi Equations and Viscosity solutions.
5. Applications to Machine Learning and Artificial Intelligence.