PhD COURSES 2026/27

Titles

1. **An introduction to Γ-convergence and applications**, 10h
2. **Introduction to Optimal Transport and Applications**, 16 h
3. **Introduction to Quantum Computing**, 14h
4. **Mathematical models for economic equilibria**, 10h
5. **Perturbation Methods for the Stability Analysis of Dynamical Systems**, 10h
6. **Selected topics in Algebra**, 10 h
7. **Selected topics in Geometry**, 10h
8. **Selected topics in Mathematical Physics**, 10 h
9. **Selected topics in Probability**, 10h
10. **Selected topics in Numerical Analysis**, 10h
11. **Towards Metamaterials: Variational methods in continuum mechanics**, 10h

Abstracts

1. **An introduction to Γ-convergence and applications**, 10h

   A wide class of mathematical models can be formulated as variational principles, where the associated optimization problems involve energy functionals. A central issue in the analysis of such models is the study of the asymptotic behavior of minima and minimizers. This motivates the introduction of Γ-convergence as a notion of convergence for functionals.

   The aim of this course is to introduce the basic theory of Γ-convergence, including compactness results, convergence of minima and minimizers, and standard methods for computing Γ-limits. The second part of the course illustrates some classical applications in phase transition and homogenization problems.

2. **Introduction to Optimal Transport and Applications**, 16h

   The theory of optimal transport dates back to Gaspard Monge (1781), who proposed the problem of minimising the cost of transporting an amount of material from a given source set to a target set of destination. In the forties, Kantorovich made a fundamental progress by reformulating the problem in such a way to provide solutions and study them. Since then, the Monge-Kantorovitch problem has been a classical subject in probability theory, economics, and optimisation. During the course, we shall discuss the existence and the properties of the optimal map or plan in Monge and Kantorovich's problems, under different conditions on the cost. We shall exploit the relation with Kantorovich's duality theorem and with Brenier's polar decomposition theorem. Furthermore, we will introduce the so-called Wasserstein distance between probability measures. In the last part of the course, we shall focus on some applications of the optimal transport theory, for instance in the study of PDEs in metric spaces, relevant in biology, data science, and opinion formation.

3. **Introduction to Quantum Computing**, 14h

   Structure of the course: 8 hours of lectures + 6 hours of computer lab hands-on tutorial

   The present short course is a joint PhD course between the PhD in Mathematics and Models, the PhD in Physics and Chemistry and the PhD in Information and Communication Technology.

The aim of the course is to provide students with different background basic knowledge of quantum computation, both from the theoretical and the computational side. The course will consist in theoretical lectures as well as hands-on tutorial with practical exercise on Python simulators and real quantum devices in the IBM-Q cloud.

Topics: General overview on quantum computation. Introduction to Quantum Mechanics and Qubits. Quantum circuits and algorithms. Single and double Qubit gates with examples. Present and future applications. Perspective of quantum computation and practical implementation of algorithms on the IBM-Q quantum computer and simulator.

4. **Mathematical models for economic equilibria**, 10h

   In science the term *equilibrium* has been widely used in physics, chemistry, biology, engineering and economics, among others, within different frameworks. It generally refers to conditions or states of a system in which all competing influences are balanced.

   For instance, the economic equilibrium which studies the dynamics of supply, demand, and prices in an economy within several markets, can be modeled as a variational inequality problem. In non-cooperative game involving two or more players, Nash proposed an equilibrium solution in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy. This problem can be reformulated as a fixed point problem.

   These mathematical models share an underlying common structure that allows to conveniently formulate them in a unique format of equilibrium. The course is devoted to describe this format and it focuses on the main mathematical tools which are crucial for studying the existence and the stability of the solutions.

5. **Perturbation Methods for the Stability Analysis of Dynamical Systems**, 10h

   The course introduces the fundamentals of perturbation analysis for linear and nonlinear dynamical systems, with particular emphasis on eigenvalues and eigenvectors sensitivity analysis and on the Multiple Scale Method for ordinary differential equations.

   The main topics include eigenvalues and eigenvectors sensitivity analysis, initial value problems solved through straightforward expansions, and the Multiple Scale Method. Applications to linear and nonlinear stability analysis are then discussed, with a focus on mechanical systems interacting with steady wind flows or subjected to follower forces. These examples illustrate how perturbation methods can be used to investigate the onset of instability, describe post-critical behavior, and assess the role of nonlinear effects.

6. **Selected topics in Algebra**, 10h

   The aim of this course is to present selected research topics investigated by the Algebra Group of the Department of Information Engineering, Computer Science and Mathematics. In particular, the topics addressed are related to brace theory, permutation group theory, near-ring theory, and the interconnections among these areas. Braces were introduced by Rump with the aim of applying ring-theoretic and group-theoretic techniques to the study of non-degenerate involutive set-theoretical solutions of the Yang-Baxter equation arising in mathematical physics. One approach to the study of these algebraic structures consists in analyzing regular subgroups of the holomorph group, or in defining such structures starting from a near-ring of functions.

7. **Selected topics in Geometry**, 10h

   The course aims to present a selection of advanced geometry topics; the choice of topics covered as well as the reference teacher varies from year to year in order to have a very wide spectrum of offerings with modern and frontier topics. The choice of topics can also be made based on the students who will follow the course.

8. **Selected topics in Mathematical Physics**, 10h

   The course aims to present a selection of advanced topics in mathematical physics; the choice of topics covered and the reference teacher varies from year to year to have a vast spectrum of offerings with modern and frontier topics. The choice of topics can also be made based on the students who will attend the course.

9. **Selected topics in Probability**, 10h

   The course aims to present a selection of advanced topics in probability; the choice of topics covered and the reference teacher varies from year to year to have a vast spectrum of offerings with modern and frontier topics. The choice of topics can also be made based on the students who will attend the course.

10. **Selected topics in Numerical Analysis**, 10h

    The course aims to present a selection of advanced topics in numerical analysis; the choice of topics covered and the reference teacher varies from year to year to have a vast spectrum of offerings with modern and frontier topics. The choice of topics can also be made based on the students who will attend the course.

11. **Towards Metamaterials: Variational methods in continuum mechanics**, 10h

    1. Principle of Virtual Work as a fundamental postulate for mechanics Second Gradient Continuum Mechanics. Hamilton-Rayleigh Principle for dissipative systems.
    2. Generalisation of the concept of Deformation and Stress: Necessary strong form for Equilibrium Conditions. Essential and Natural Boundary Conditions.
    3. Piola Transformations and contact interactions for Second Gradient Continua.
    4. Edge and Surface contact interactions in second gradient continua: forces and double forces. Representation of contact interactions in terms of stresses, double stresses and shape of Cauchy cuts Limitations of so called Cauchy postulate.
    5. Some remarks on relevant aspects of the history of mechanics and in particular on the development of the concepts of force, stress and couples.