# PhD COURSES 2026/27

## Titles

1. **An introduction to $\Gamma$-convergence and applications**, 10h, M. Caponi
2. **Introduction to Optimal Transport and Applications**, 16 h, A. Esposito, S. Fagioli
3. **Introduction to Quantum Computing**, 14h, L. Guidoni
4. **Mathematical models for economic equilibria**, 10h, M. Giuli
5. **Perturbation Methods for the Stability Analysis of Dynamical Systems**, 10h, M. Ferretti
6. **Selected topics in Algebra: Introduction to Matroid Theory**, 10h, S. Di Trani
7. **Selected topics in Geometry**, 10h, M. Santilli
8. **Selected topics in Mathematical Physics: Scientific Machine Learning**, 10 h, G. Arampatzis
9. **Selected topics in Probability: From classical to functional and stable central limit theorems**, 10h, I.G. Minelli
10. **Selected topics in Numerical Analysis**, 10h, R. D'Ambrosio
11. **Towards Metamaterials: Variational methods in continuum mechanics**, 10h, A. Ciallella, F. Dell'Isola
12. **PDE Modelling of Transportation Networks**, 10h, P. Markowich
13. **From particles to Earth's crust: AI and statistical mechanics methods in geophysical processes**, 10h, F. Di Michele, M. Colangeli

## Abstracts

1. **An introduction to $\Gamma$-convergence and applications**, 10h

   **Lecturer:** Maicol Caponi

   A wide class of mathematical models can be formulated as variational principles, where the associated optimization problems involve energy functionals. A central issue in the analysis of such models is the study of the asymptotic behavior of minima and minimizers. This motivates the introduction of $\Gamma$-convergence as a notion of convergence for functionals.

   The aim of this course is to introduce the basic theory of $\Gamma$-convergence, including compactness results, convergence of minima and minimizers, and standard methods for computing $\Gamma$-limits. The second part of the course illustrates some classical applications in phase transition and homogenization problems.

2. **Introduction to Optimal Transport and Applications**, 16h

   **Lecturers:** Antonio Esposito, Simone Fagioli

   The theory of optimal transport dates back to Gaspard Monge (1781), who proposed the problem of minimising the cost of transporting an amount of material from a given source set to a target set of destination. In the forties, Kantorovich made a fundamental progress by reformulating the problem in such a way to provide solutions and study them. Since then, the Monge-Kantorovitch problem has been a classical subject in probability theory, economics, and optimisation. During the course, we shall discuss the existence and the properties of the optimal map or plan in Monge and Kantorovich's problems, under

different conditions on the cost. We shall exploit the relation with Kantorovich's duality theorem and with Brenier's polar decomposition theorem. Furthermore, we will introduce the so-called Wasserstein distance between probability measures. In the last part of the course, we shall focus on some applications of the optimal transport theory, for instance in the study of PDEs in metric spaces, relevant in biology, data science, and opinion formation.

3. **Introduction to Quantum Computing**, 14h,

   **Lecturer:** Leonardo Guidoni

   Structure of the course: 8 hours of lectures + 6 hours of computer lab hands-on tutorial

   The present short course is a joint PhD course between the PhD in Mathematics and Models, the PhD in Physics and Chemistry and the PhD in Information and Communication Technology.

   The aim of the course is to provide students with different background basic knowledge of quantum computation, both from the theoretical and the computational side. The course will consist in theoretical lectures as well as hands-on tutorial with practical exercise on Python simulators and real quantum devices in the IBM-Q cloud.

   Topics: General overview on quantum computation. Introduction to Quantum Mechanics and Qubits. Quantum circuits and algorithms. Single and double Qubit gates with examples. Present and future applications. Perspective of quantum computation and practical implementation of algorithms on the IBM-Q quantum computer and simulator.

4. **Mathematical models for economic equilibria**, 10h

   **Lecturer:** Massimiliano Giuli

   In science the term *equilibrium* has been widely used in physics, chemistry, biology, engineering and economics, among others, within different frameworks. It generally refers to conditions or states of a system in which all competing influences are balanced.

   For instance, the economic equilibrium which studies the dynamics of supply, demand, and prices in an economy within several markets, can be modeled as a variational inequality problem. In non-cooperative game involving two or more players, Nash proposed an equilibrium solution in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy. This problem can be reformulated as a fixed point problem.

   These mathematical models share an underlying common structure that allows to conveniently formulate them in a unique format of equilibrium. The course is devoted to describe this format and it focuses on the main mathematical tools which are crucial for studying the existence and the stability of the solutions.

5. **Perturbation Methods for the Stability Analysis of Dynamical Systems**, 10h

   **Lecturer:** Manuel Ferretti

   The course introduces the fundamentals of perturbation analysis for linear and nonlinear dynamical systems, with particular emphasis on eigenvalues and eigenvectors sensitivity analysis and on the Multiple Scale Method for ordinary differential equations.

   The main topics include eigenvalues and eigenvectors sensitivity analysis, initial value problems solved through straightforward expansions, and the Multiple Scale Method. Applications to linear and nonlinear stability analysis are then discussed, with a focus on mechanical systems interacting with steady wind flows or subjected to follower forces. These examples illustrate how perturbation methods can be used to investigate the onset of instability, describe post-critical behavior, and assess the role of nonlinear effects.

6. **Selected topics in Algebra: Introduction to Matroid Theory**, 10h

   **Lecturer**: Sabino Di Trani

   Matroids are combinatorial structures that arise in many areas, including linear algebra, graph theory, combinatorial geometry, and optimization. They were introduced to generalize the concept of linear independence and provide a unified framework for studying a wide range of problems in combinatorics and computer science.

   Throughout the course, we will explore the general features of matroid theory, while also discussing concrete examples and applications in combinatorics, topology, and computer science.

   Particular attention will be given to graphic matroids and their connections with algebraic invariants of graphs. We will also introduce the Tutte polynomial of a matroid and focus on the rich combinatorial and algebraic information it encodes.

7. **Selected topics in Geometry**, 10h

   **Lecturer**: Mario Santilli

   The course aims to present a selection of advanced geometry topics; the choice of topics covered as well as the reference teacher varies from year to year in order to have a very wide spectrum of offerings with modern and frontier topics. The choice of topics can also be made based on the students who will follow the course.

8. **Selected topics in Mathematical Physics**, 10h

   **Title:** Scientific Machine Learning: Bridging Differential Equations and Data-Driven Methods

   **Lecturer:** Georgios Arampatzis (University of Crete)

   **Abstract:** At the intersection of physics-based modeling and modern machine learning lies Scientific Machine Learning (SciML): a growing discipline that applies neural networks to problems governed by differential equations, conservation laws, and other mathematical structure. By embedding physical knowledge into neural architectures, SciML produces models that are both expressive enough to handle complex data and grounded enough to be trusted for scientific inference. This lecture series provides a rigorous yet accessible entry point into its foundational methods and computational tools.

   The first lecture establishes the dynamical systems foundations - discrete and continuous - that motivate the SciML approach, along with their classical numerical treatment. We then introduce Physics-Informed Neural Networks (PINNs), which embed governing differential equations directly into the training loss, enabling data-efficient learning in regimes where observations are sparse or noisy.

   The second lecture focuses on the computational infrastructure underpinning modern SciML. We cover automatic differentiation as a first-class scientific tool, the adjoint method for gradient computation through differential equation solvers, and Neural ODEs - models that represent the evolution of a hidden state as a differential equation, bridging the architectures of deep learning with the language of dynamical systems. Practical implementation is carried out in PyTorch, with attention to vectorization and performance.

   The third lecture moves to operator learning, presenting DeepONets and Neural Operators as frameworks for learning mappings between function spaces - a major step beyond pointwise regression. We close with inverse problems and their treatment through a Bayesian lens, connecting uncertainty quantification to the broader SciML workflow.

   Throughout, emphasis is placed on implementation and physical intuition. Participants will encounter concrete examples drawn from computational physics, applied mathematics, and engineering, illustrating both the promise and the current limitations of the field. For those wishing to deepen their understanding, hands-on programming problems will be distributed, covering the key methods presented across the three lectures

9. **Selected topics in Probability: From classical to functional and stable central limit theorems**, 10h

   **Lecturer:** Ida Germana Minelli

   The course explores extensions of the classical central limit theorem to random processes and to settings with random limiting distributions. Starting with a review of characteristic functions and the multivariate CLT, we introduce weak convergence of probability measures on spaces of continuous functions, tightness, and Donsker's invariance principle. We then discuss stable convergence and its role in central limit theorems with random asymptotic variances, including functional versions. Selected examples and applications will illustrate the main ideas.

10. **Selected topics in Numerical Analysis**, 10h

    **Lecturer:** Raffaele D'Ambrosio

    The course aims to present a selection of advanced topics in numerical analysis; the choice of topics covered and the reference teacher varies from year to year to have a vast spectrum of offerings with modern and frontier topics. The choice of topics can also be made based on the students who will attend the course.

11. **Towards Metamaterials: Variational methods in continuum mechanics**, 10h

    **Lecturers:** Alessandro Ciallella, Francesco Dell'Isola

    1. Principle of Virtual Work as a fundamental postulate for mechanics Second Gradient Continuum Mechanics. Hamilton-Rayleigh Principle for dissipative systems.
    2. Generalisation of the concept of Deformation and Stress: Necessary strong form for Equilibrium Conditions. Essential and Natural Boundary Conditions.
    3. Piola Transformations and contact interactions for Second Gradient Continua.
    4. Edge and Surface contact interactions in second gradient continua: forces and double forces. Representation of contact interactions in terms of stresses, double stresses and shape of Cauchy cuts Limitations of so called Cauchy postulate.
    5. Some remarks on relevant aspects of the history of mechanics and in particular on the development of the concepts of force, stress and couples.

12. **PDE Modelling of Transportation Networks**, 10h

    **Lecturer:** Peter Markowich (KAUST, Accademia Nazionale dei Lincei)

    This course will give an overview of recent partial differential equation models for the emergence, formation and evolution of biological transport networks. Typically, the models describe the material pressure field using a Darcy's type equation and the dynamics of the network conductance. Randomness in the porous medium material structure is represented by linear diffusion and conductance relaxation by an algebraic decay term.

    Micro- and mesoscopic models will be introduced and will show how they are connected to the classical macroscopic PDE system (by D. Hu and D. Cai) and to discrete graph based models.

13. **From particles to Earth's crust: AI and statistical mechanics methods in geophysical processes**, 10h:

    **Lecturers:** Federica Di Michele (INGV), Matteo Colangeli

    The course introduces the fundamental concepts of Machine Learning, its core models, and advanced neural network architectures. Particular emphasis will be placed on Physics-Informed Neural Networks (PINNs) and their application to geophysical and seismological problems. Additionally, the course will outline statistical mechanical models and methods, bridging the gap between microscopic particle dynamics and large-scale geophysical processes.