# Divergence-free and divergence-controlled positive symmetric tensors. Applications to mathematical physics

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**Abstract**

We present a new aspect of functional analysis, where the objects are positive semi-definite symmetric tensors, whose row-wise divergence either vanishes or takes values in bounded measures. Qualitatively, the determinant has an unexpected integrability property. Quantitatively, the corresponding integral obeys a sharp estimate. This is a versatile tool, which has application in several domains in mathematics or mathematical physics.

For instance, we recover the isoperimetric inequality, and we have a new characterization of minimal surfaces. We shall investigate at depth applications to gas dynamics at every level of description: macroscopic (Euler equations), mesoscopic (Boltzmann) and microscopic (molecular, or hard spheres dynamics). We shall also present applications to plasmas (Vlasov-Poisson) and explain why the tool is not suitable for viscous fluids (Navier-Stokes) and for the dynamics of galaxies; the fact is that the positivity of the tensor occurs when the force at stake is repulsive. Eventually, we explain how it can be used in the study of multi-dimensional conservation laws, to obtain dispersion relations, and to establish the existence of solutions for some singular initial data.

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