

## PHD COURSES 2021/2022

**Apart from specific situations due to the pandemic that may change the organization of some courses, all the courses will be held in presence in classroom 0.6 of the building "Coppito 1" and will also be streamed. Please refer to the details indicated below for each course.**

### MANDATORY COURSES

- **(A1)** Signed graphs, gain graphs and their spectral theory:  
**Matteo Cavaleri**, University Niccolò Cusano, Roma (12 hours)

PROGRAM: See attached file

TECHNICAL INFORMATION: Interested students should write to [matteo.cavaleri@unicusano.it](mailto:matteo.cavaleri@unicusano.it) to have information about the lectures (google meet).

- **(FM1)** Foundations of nonequilibrium statistical mechanics: mathematical tools and physics:  
**Lamberto Rondoni**, Politecnico di Torino (6 hours)

PROGRAM: Three two-hours lectures, for a total of six hours. Almost each point listed below would require six hours, if approached in detail. The ergodic theorem would require more by itself. The purpose of these lectures is thus to mention interesting research lines, and to bring to the fore potentiality and obstacles in applying certain mathematical techniques to the physics of nonequilibrium phenomena.

Content of the Lectures:

Lecture 1. Ergodic theory: motivation, fundamental definitions, ergodic theorem of Birkhoff and the bottleneck on the road to applications. Khinchin's ergodic theorem.

Lecture 2. Probability: physics theories provide a framework for the interpretation and understanding of quantitative observations of material objects, in the sense that it enjoys universality and predictive value. Probability is most useful in this endeavour. Per se, it is "immaterial", but it turns concrete, "material", under numerous but often verified conditions. The Boltzmann equation.

Lecture 3. Ergodic notions for non-invariant measures: fluctuation theorem, dissipation function, response theory, t-mixing and the bottleneck. Applications to molecular dynamics. Kuramoto: an exactly solvable model.

TECHNICAL INFORMATION: Teams channel "Nonequilibrium Statistical Mechanics" (code: b1ibgu6). Reference e-mail [lamberto.rondoni@polito.it](mailto:lamberto.rondoni@polito.it)

- **(FM2)** Why the microscopic world is not "classic"?:  
**Maurizio Serva**, UnivAQ (4 hours)

PROGRAM: This extremely short course will not explain what Quantum Mechanics is. To tell the truth, it can't even be considered a real course but it is more an attempt to arouse your curiosity about the laws that govern the microscopic world. We will start from one of the most iconic experimental evidences that has no explanation in the context of Classical Mechanics and then we will present some elements of the new theory together with some apparently paradoxical consequences.

Content of the lectures:

Lecture 1. Diffraction of electrons: the double-slit experiment.

Lecture 2. Wave function and probability. The superposition principle.

Lecture 3. Hermitian operators and observables.

Lecture 4. The Heisenberg's uncertainty principle.

TECHNICAL INFORMATION: Reference e-mail [maurizio.serva@univaq.it](mailto:maurizio.serva@univaq.it)

- **(SD1)** Perturbation Methods for the Stability Analysis of Dynamical Systems:

**Simona Di Nino**, UnivAQ (6 hours)

PROGRAM: See the attached file

TECHNICAL INFORMATION: Teams channel *Perturbation Methods for the Stability Analysis of Dynamical Systems*, code yqa61wb. Reference e-mail simona.dinino@univaq.it

• **(An1)** Operator semigroups and applications:

**Klaus Engel**, UnivAQ (6 hours)

PROGRAM: The theory of one-parameter semigroups of bounded linear operators on a Banach space provides a powerful tool to study, in a systematic and unified way, the well-posedness of a wide range of linear evolution equations. Moreover, it allows to obtain detailed information about the qualitative properties of the solutions like long-time behavior or positivity.

The aim of this short course, consisting of 3 lectures, is to give a brief introduction into this subject and to sketch some application. To this end the first part recalls the necessary notions from functional analysis and operator theory. In the second part the basic results of the abstract theory are presented and illustrated by some standard examples. Finally, these abstract results are applied to study the well-posedness and the qualitative properties of the solutions of some concrete evolution equation.

References (cf. [www.math.uni-tuebingen.de/de/forschung/agfa/members/rana](http://www.math.uni-tuebingen.de/de/forschung/agfa/members/rana))

\* K.-J. Engel, R. Nagel. *One-Parameter Semigroups for Linear Evolution Equations*, Graduate Texts in Mathematics, vol. 194. Springer-Verlag, 2000.

or (short version, less applications)

\* K.-J. Engel, R. Nagel. *A Short Course on Operator Semigroups*, Universitext. Springer-Verlag, 2006.

TECHNICAL INFORMATION: Teams code: **rqzb3yp** *PhD Courses - Mathematical Analysis - 2021/22*

• **(An2)** Introduction to Optimal Control:

**M. Palladino** and **T. Scarinci**, UnivAQ (12 hours)

PROGRAM: The course aims at providing an overview of optimal control theory, with a special emphasis on dynamic constraint described by a differential inclusion  $F(t, x)$ . This is a very general case since it covers the standard optimal control framework in which the dynamic constraint is a controlled differential equation as well as many others.

The main course topics that will be covered are: Introduction to Differential Inclusions, Filippov Existence Theorem; Necessary conditions for optimality; Invariance theory and derivation of Hamilton-Jacobi Equation.

TECHNICAL INFORMATION: Teams code: **rqzb3yp** *PhD Courses - Mathematical Analysis - 2021/22*

• **(An3)** Weak derivatives and beyond:

**Raffaella Giova**, Uniparthenope (6 hours)

PROGRAM: Weak derivatives Singularity at one point Sobolev spaces Weak derivatives improve the integrability of the function Absolute continuity on lines Bounds in  $L^p$  for the difference quotient Fractional weak derivatives Fractional weak derivatives improve the integrability of the function Applications Derivatives in metric spaces

References:

- Campanato Sergio, *Sistemi ellittici in forma divergenza. Regolarità all'interno*. Quaderni Scuola Norm. Sup. Pisa 1980

- Giaquinta Mariano, *Multiple integrals in the calculus of variations and nonlinear elliptic systems*. Princeton University Press, Princeton 1983

- Giusti Enrico, *Equazioni ellittiche del secondo ordine*. Quaderno U.M.I. no. 6, Pitagora Ed. 1978

- Kufner Alois, John Oldřich, Fučík Svatopluk, *Function spaces. Monographs and Textbooks on Mechanics of Solids and Fluids, Mechanics: Analysis*. Noordhoff International Publishing, Leyden; Academia, Prague, 1977

- Hajłasz Piotr, *Sobolev spaces on an arbitrary metric space*. Potential Anal. 5 (1996), no. 4, 403–415.

- Leonetti Francesco, Dispense del corso di analisi superiore.
- Nirenberg Louis, Remarks on strongly elliptic partial differential equations. *Comm. Pure Appl. Math.* 8 (1955), 649–675

TECHNICAL INFORMATION: Teams code: **rqzb3yp** *PhD Courses - Mathematical Analysis - 2021/22*

- **(N1)** Introduction to the Finite Element Method for Partial Differential Equations:  
**Raffaele D'Ambrosio**, UnivAQ (10 hours)

PROGRAM: This is an introductory course on FEM discretizations to selected partial differential equations. The idea is to proceed by classes of problems, identifying the most appropriate numerical solvers. This type of approach is designed for PhD students involved in research areas where the numerical treatment of PDEs is required. Tentative list of topics:

- FEM for elliptic problems.
- FEM for parabolic problems.
- FEM for hyperbolic problems.
- 1D e 2D grid generation.
- Spectral methods.
- Discontinuous Galerkin.

TECHNICAL INFORMATION: Teams channel with code s32ahf6. Reference e-mail [raffaele.dambrosio@univaq.it](mailto:raffaele.dambrosio@univaq.it)

- **(Q1)** Quantum Computing:  
**Leonardo Guidoni**, UnivAQ + Expert from IBM (14 hours)

PROGRAM: See the attached file.

TECHNICAL INFORMATION: Teams channel with code aw13okq Reference e-mail [leonardo.guidoni@univaq.it](mailto:leonardo.guidoni@univaq.it)

- **(P1)** Combinatorial species, Graph generating functions and statistical mechanics:  
**Dimitrios Tsagkarogiannis** UnivAQ (6 hours)

PROGRAM: See the attached file.

TECHNICAL INFORMATION: Teams channel xd7j0by Reference e-mail [dimitrios.tsagkarogiannis@univaq.it](mailto:dimitrios.tsagkarogiannis@univaq.it)

- **(E1)** Mathematical models for economic equilibria:  
**Massimiliano Giuli**, UnivAQ (10 hours)

PROGRAM: See the attached file.

TECHNICAL INFORMATION: Teams channel -Mathematical models for economic equilibria- with code ooye5vv Reference e-mail [massimiliano.giuli@univaq.it](mailto:massimiliano.giuli@univaq.it)

- **(G1)** Constant mean curvature hypersurfaces and critical points of the isoperimetric problem:  
**Mario Santilli**, UnivAQ (10 hours)

PROGRAM: See the attached file

TECHNICAL INFORMATION: Lectures on webex, write to [mario.santilli@univaq.it](mailto:mario.santilli@univaq.it) for information

- **(V1)** Variational methods in continuum mechanics:  
**Dell'Isola-Ciallella**, UnivAQ (10 hours)

PROGRAM: 1. Principle of Virtual Work as a fundamental postulate for mechanics Second Gradient Continuum Mechanics. Hamilton Rayleigh Principle for dissipative systems 2. Generalisation of the concept of Deformation and Stress: Necessary strong form for Equilibrium Conditions Essential and Natural Boundary Conditions 3. Piola Transformations and contact interactions for Second Gradient Continua 4. Edge and Surface contact interactions in second gradient continua: forces and double forces. Representation of contact interactions in terms of stresses, double stresses and shape of Cauchy cuts Limitations of so called Cauchy

postulate 5. Some remarks on relevant aspects of history of mechanics and in particular on the development of the concepts of force, stress and couples.

References:

Spagnuolo, Mario, Francesco dell'Isola and Antonio Cazzani. "The study of the genesis of novel mathematical and mechanical theories provides an inspiration for future original research." *Evaluation of Scientific Sources in Mechanics*. Springer, Cham, 2022. 1-73.

Eugster, S. R., Dell'Isola, F., Fedele, R., and Seppecher, P. (2021). Piola Transformations in Second Gradient Continua.

Auffray, N., dell'Isola, F., Eremeyev, V. A., Madeo, A., and Rosi, G. (2015). Analytical continuum mechanics à la Hamilton–Piola least action principle for second gradient continua and capillary fluids. *Mathematics and Mechanics of Solids*, 20(4), 375-417.

dell'Isola, F., Seppecher, P., Placidi, L., Barchiesi, E., and Misra, A. (2020). Least action and virtual work principles for the formulation of generalized continuum models. *Discrete and Continuum Models for Complex Metamaterials*, 327.

TECHNICAL INFORMATION: Reference e-mail francesco.dellisola@univaq.it

## OTHER COURSES

### **(A2) Application of abelian regular groups to differential cryptanalysis**

**Roberto Civino**, UnivAQ (4 hours)

PROGRAM: Is it possible that a block cipher apparently immune to classical differential cryptanalysis can be attacked considering a different operation on the message space? In this course we answer to this question investigating how alternative operations interact with the layers of a substitution–permutation network and showing how they influence the differential probabilities, when the difference taken into consideration is different from the usual bit-wise addition modulo two. Furthermore, we design a block cipher which appears to be secure with respect to classical differential cryptanalysis, but weaker with respect to our attack which makes use of alternative operations.

TECHNICAL INFORMATION: reference e-mail roberto.civino@univaq.it

### **N2 Inverse Problems in Signal Processing: new results and open challenges:**

**Antonio Cicone**, UnivAQ (12 hours)

PROGRAM: One classical problem in many applied fields of research, like Geophysics, Medicine, Engineering, Economy and Finance, is, given a signal, how to extract hidden information and features contained in it, like, for instance, quasiperiodicities and trends. Standard methods like Fourier and Wavelet Transform, historically used in signal processing, proved to be limited when nonlinear and nonstationary phenomena are present. For this reason in the last two decades several new nonlinear methods have been developed by many research groups around the world and they have been used extensively in the applications. In this course we will review the Empirical Mode Decomposition method and derived techniques, and introduce the Iterative Filtering technique and its generalizations. We will discuss their theoretical and numerical properties, show their limitations and discuss open problems. During the course some applications will be presented.

TECHNICAL INFORMATION: Reference e-mail antonio.cicone@univaq.it

**Signed graphs, gain graphs and their spectral theory.**  
Matteo Cavaleri, *Università degli studi Niccolò Cusano*, Roma

Graphs are ubiquitous in many fields of mathematics. The same is true for their recent generalizations, namely signed and gain graphs. First introduced to handle problems in social psychology, signed graphs are graphs whose edges can be positive or negative. More generally, gain graphs are graphs where each oriented edge is labeled by an element of a group  $G$  in such a way that the opposite orientation corresponds to the group inverse of the element.

Spectral graph theory is the study of these combinatorial objects, graphs, via linear algebra. In fact, several complex matrices, most of which Hermitian, can be associated to a graph. Some important properties of a graph can be detected by the real spectrum of the corresponding matrix. The generalization of this theory to signed graphs is very natural, since Hermitian matrices are naturally associated with a signed graph. Nevertheless this generalization is far from being trivial, because on the one hand it reflects peculiar properties of signed graphs, and on the other hand many questions are still open [3].

For general gain graphs a crucial obstruction to develop a spectral theory is that the matrices that come out with a gain graph on an abstract group  $G$  are not complex matrices, but group algebra valued matrices. A possible strategy is to use the group representation theory to study the spectrum of a gain graph with respect to a unitary representation  $\pi$ , via Fourier transforms. This way, many spectral results that hold for graphs and signed graphs can be generalized to gain graphs.

The aim of this course is to discuss generalizations of graph theory, with a special focus on spectral aspects, to signed and gain graphs. These generalizations will be presented after introducing some basic notions and facts about (finite, simple) graphs.

In more detail:

- Preliminaries on graphs (4h)
  - Basic notions, isomorphism, regularity, bipartiteness, subgraphs, walks, trees and spanning trees, cycle basis, 2-cover.
  - Adjacency matrix, Laplacian matrix, incidence matrix. Spectrum, fundamental theorems about spectrum (*which combinatorial properties of the graph can be deduced from the algebraic invariants of the graph matrix?*). Interlace theorem, Sach's formula (pointers, [7]).
  - Cospectrality (*which combinatorial properties of the graph can not be deduced from the algebraic invariants of the graph matrix?*), Godsil-McKay switching [8].
  - Line graphs: definition, relation with incidence matrices, spectrum. Whitney isomorphism theorem (no proof), Beineke's, Krausz's, Rooij and Wilf's characterizations (Harary's proof [11]).
- Signed Graphs (4h)

- Basic notions and motivation (model of positive/negative interactions in a network). Balance (Harary’s characterizations [10]). Cover graph, pointers to voltage graphs [9].
- Adjacency matrix, spectrum. Switching equivalence and switching isomorphism [13]. Sign on a cycle basis. Spectral radius, sign-symmetric signed graphs. Acharya’s spectral characterization of balance signed graphs.
- Cospectrality, generalization of Godsil-McKay switching for signed graphs [1]. Hints on Godsil-McKay switching for complex unit graphs [2].
- Signed line graphs according to Zaslavsky [14, 15]. Spectrum. Anticipation of the spectral characterization theorem for signed line graphs with underlying graph that is a line graph [6].
- Gain Graphs (8h)
  - Definition and basic notions. Balance, switching equivalence and switching isomorphism, cover graph of a gain graph.
  - Preliminaries on group algebra and group algebra valued matrices. Abstract adjacency matrix of a gain graph and its properties. Characterization of balance in terms of the trace of increasing powers of the abstract adjacency matrix of a gain graph [4].
  - Preliminaries on unitary representations  $\pi$  of a group  $G$ . Fourier transforms: from group algebra to group algebra valued matrices. Application to the computation of the spectrum of a  $G$ -block circulant matrix ([12], proof from [4]).
  - $\pi$ -adjacency matrix and  $\pi$ -spectrum of a gain graph. Generalization of Acharya’s spectral characterization to gain graphs [4, Th. 5.1]. Decomposition of the spectrum of a cover graph (proof from [4]).
  - Set of  $G$ -phases and their associated actions. Gain-line graph.  $\pi$ -spectrum of a gain-line graph [5].
  - Generalization of Beineke’s, Krausz’s, Rooij and Wilf’s characterizations to gain-line graphs [6, Th. 3.2]. The special case of signed graphs, spectral consequences.

## REFERENCES

- [1] F. Belardo, M. Brunetti, M. Cavaleri, A. Donno, Constructing cospectral signed graphs, *Linear Multilinear Algebra* (2019) DOI: 10.1080/03081087.2019.1694483
- [2] F. Belardo, M. Brunetti, M. Cavaleri, A. Donno, Godsil-McKay switching for mixed and gain graphs over the circle group, *Linear Algebra Appl.* **614** (2020), 256–269.
- [3] F. Belardo, S. M. Cioabă, J.H. Koolen, J. Wang, Open problems in the spectral theory of signed graphs, available on arXiv:1907.04349.
- [4] M. Cavaleri, D. D’Angeli, A. Donno, A group representation approach to balance of gain graphs, *J. Algebr. Combin.* **54** (2021), no. 1, 265–293.
- [5] M. Cavaleri, D. D’Angeli, A. Donno, Gain-line graphs via  $G$ -phases and group representations, *Linear Algebra Appl.* **613** (2021), 241–270.
- [6] M. Cavaleri, D. D’Angeli, A. Donno, Characterizations of line graphs in signed and gain graphs, available on arXiv:2101.09677.

- [7] D. Cvetković, P. Rowlinson, S. Simić, *An Introduction to the Theory of Graph Spectra*, London Mathematical Society Student Texts, **75**, Cambridge University Press, Cambridge, 2010, xii+364 pp.
- [8] C.D. Godsil, B. McKay, Constructing cospectral graphs, *Aequationes Math.* **25** (1982), 257–268.
- [9] J.L. Gross, T.W. Tucker, Generating all graph coverings by permutation voltage assignments, *Discrete Math.* **18**, no. 3, 273–283 (1977).
- [10] F. Harary, On the notion of balance of a signed graph, *Michigan Math. J.* **2** (1953–1954), 143–146 (1955).
- [11] F. Harary, *Graph theory*, Addison-Wesley Publishing Co., Reading, Mass.-Menlo Park, Calif.-London 1969, ix+274 pp.
- [12] G.J. Tee, Eigenvectors of block circulant and alternating circulant matrices, *New Zealand J. Math.* **36**, 195–211 (2007).
- [13] T. Zaslavsky, Characterizations of signed graphs. *J. Graph Theory* **5** (1981), no. 4, 401–406.
- [14] T. Zaslavsky, Line graphs of switching classes, *Report of the XVIIIth O.S.U. Denison Maths Conference* (Granville, Ohio, 1984), pp. 2–4. Dept. of Math., Ohio State Univ., Columbus, Ohio, 1984.
- [15] T. Zaslavsky, Matrices in the theory of signed simple graphs. *Advances in discrete mathematics and applications: Mysore, 2008*, 207–229, Ramanujan Math. Soc. Lect. Notes Ser. **13**, Ramanujan Math. Soc., Mysore, 2010.

# Mathematical models for economic equilibria (10 hours)

Massimiliano Giuli

In science the term “equilibrium” has been widely used in physics, chemistry, biology, engineering and economics, among others, within different frameworks. It generally refers to conditions or states of a system in which all competing influences are balanced.

For instance, the economic equilibrium which studies the dynamics of supply, demand, and prices in an economy within several markets, can be modeled as a variational inequality problem. In non-cooperative game involving two or more players, Nash proposed an equilibrium solution in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy. This problem can be reformulated as a fixed point problem.

These mathematical models share an underlying common structure that allows to conveniently formulate them in a unique format of equilibrium. The course is devoted to describe this format and it focuses on the main mathematical tools which are crucial for studying the existence and the stability of the solutions.



# Constant mean curvature hypersurfaces and critical points of the isoperimetric problem

Mario Santilli

November 24, 2021

The aim of this course is to give a detailed overview on some classical and recent results in the theory of constant mean curvature hypersurfaces. In this field a fundamental result is a theorem of Alexandrov (1958), which asserts that a compact embedded smooth hypersurface in the Euclidean (or hyperbolic) space with constant scalar mean curvature is a (geodesic) sphere. This result was the starting point of a line of research that is still very active. In this course, besides recalling the classical proof of Alexandrov based on the celebrated "moving-plane method", we give a detailed overview of some of the most significant developments and we explore the strong connections with the isoperimetric problem. In particular we aim to treat the following topics:

1. Alexandrov theorem: the original proof in  $\mathbf{R}^n$  and  $\mathbf{H}^n$  (hyperbolic space) based on the "moving-plane-method" (see [Ale58]),
2. Alexandrov theorem and its extensions to higher-order mean curvatures: the PDE-approach of Ros (see [Ros87]) and the integral-geometric approach of Montiel-Ros ([MR91]),
3. Alexandrov theorem beyond the space forms: Brendle's theorem in warped product spaces (see [Bre13]),
4. Critical points of the isoperimetric problem: the Alexandrov theorem for sets of finite perimeter (see [DM19] and [DRKS20]).

**Number of hours:** 10 (5 lectures)

## References

- [Ale58] A. D. Aleksandrov. Uniqueness theorems for surfaces in the large. V. *Vestnik Leningrad. Univ.*, 13(19):5–8, 1958.
- [Bre13] Simon Brendle. Constant mean curvature surfaces in warped product manifolds. *Publ. Math. Inst. Hautes Études Sci.*, 117:247–269, 2013.
- [DM19] Matias Gonzalo Delgadino and Francesco Maggi. Alexandrov's theorem revisited. *Anal. PDE*, 12(6):1613–1642, 2019.
- [DRKS20] Antonio De Rosa, Sławomir Kolasiński, and Mario Santilli. Uniqueness of critical points of the anisotropic isoperimetric problem for finite perimeter sets. *Arch. Ration. Mech. Anal.*, 238(3):1157–1198, 2020.

- [MR91] Sebastián Montiel and Antonio Ros. Compact hypersurfaces: the Alexandrov theorem for higher order mean curvatures. In *Differential geometry*, volume 52 of *Pitman Monogr. Surveys Pure Appl. Math.*, pages 279–296. Longman Sci. Tech., Harlow, 1991.
- [Ros87] Antonio Ros. Compact hypersurfaces with constant higher order mean curvatures. *Rev. Mat. Iberoamericana*, 3(3-4):447–453, 1987.

# Combinatorial species, Graph generating functions and statistical mechanics

## Lecturer

Dimitrios Tsagkarogiannis, University of L'Aquila,  
email: dimitrios.tsagkarogiannis@univaq.it  
<https://www.disim.univaq.it/DimitriosTsagkarogiannis>

## Abstract

Informally, a combinatorial species  $F$  is a class of labelled discrete structures which is closed under isomorphisms induced by relabelling along bijections (see [1]). To each species we can associate a generating function  $F(x) = \sum_{n \geq 0} w_F(n) \frac{x^n}{n!}$ , where  $w_F(n)$  denotes the number of  $F$ -structures on the set  $[n] = \{1, 2, \dots, n\}$ . Moreover, combinatorial operations on species such as sum, product, composition and derivation, among others, have their analogues on generating functions. Equipped with this structure we can revisit Mayer's theory on non-ideal gases (see [2]) and express the key thermodynamic quantities such as pressure, free energy, correlation functions in terms of generating functions of various species (see [3]). Convergence of these series remains a key issue and we will study it using Kruskal's algorithm on graphs and Penrose's tree-graph inequality (see [4] and [5]).

## Schedule

1. **Motivation:** From partition function to graph generating functions
2. **Mayer expansion:** Combinatorial species, generating functions and related operations
3. **Convergence:** Kruskal's algorithm and Penrose's tree-graph inequality
4. **Virial expansion:** inversion using trees, Lagrange's inversion theorem, dissymmetry theorem, generating function of two-connected graphs.

## References

- [1] F. Bergeron, G. Labelle and P. Leroux, Combinatorial Species and Tree-Like Structures, Coll. Encyclopedia of mathematics and its applications, Vol. 67, Cambridge University Press, 1998.
- [2] J. E. Mayer, The Statistical Mechanics of Condensing Systems. I, J. Chem. Phys. **5**, 67 (1937).
- [3] P. Leroux, Enumerative problems inspired by Mayer's theory of cluster integrals, Electron. J. Combin. **11** (2004), paper no. R32.
- [4] D. Ueltchi, An improved tree-graph bound, arXiv:1705.05353 (2017).
- [5] O. Penrose, Convergence of fugacity expansions for fluids and lattice gases, J. Math. Phys. **4** (1963), 1312-1320.
- [6] S. Jansen, T. Kuna, D. Tsagkarogiannis: Virial inversion and density functionals, preprint arXiv:1906.02322 (2019)
- [7] S. Jansen, T. Kuna, D. Tsagkarogiannis: Lagrange inversion and combinatorial species with uncountable color palette, Ann. Henri Poincaré, pp. 1–36. Springer (2021).
- [8] E. Pulvirenti, D. Tsagkarogiannis, Cluster expansion in the canonical ensemble, *Comm. Math. Phys.* **316** (2012), no. 2, pp. 289–306.
- [9] S. J. Tate, The combinatorics of the Cluster and Virial Expansions, PhD thesis, University of Warwick (2014).

*PhD course in Mathematics and Models and PhD in Informatics*

## Introduction to Quantum Computing

Speakers:

Leonardo Guidoni (Univaq)

Hands on tutorial lead by Experts from IBM-Italia

The present short course is a joint PhD course between the PhD in Mathematics and Models and the PhD in Informatics. The aim of the short course is to provide to students with background in mathematics and informatics the foundation of quantum computation. The course will consist of theoretical lectures as well as hands-on tutorial lead by the Quantum Computing experts from IBM-Italia.

Topics: General overview on quantum computation. Introduction to Quantum Mechanics and Qubits. Quantum circuits and algorithms. Single and double Qubit gates with examples. Present and future applications. Perspective of quantum computation and practical implementation of algorithms on the IBM-Q quantum computer and simulator.

14 hours:

8 hours of lectures + 6 hours of computer lab hands-on tutorial

January 20<sup>th</sup> 11-13

January 26<sup>th</sup> 11-13

February 3<sup>rd</sup> 11-13

February 9<sup>th</sup> 11-13

February 18<sup>th</sup> 11-13

February 23<sup>rd</sup> 11-13 + 15-17

Classroom: **Aula 0.6** (Coppito 1)

Microsoft Teams Code: **aw13okq**

TITOLO DEL CORSO: Perturbation Methods for the Stability Analysis of Dynamical Systems

DOCENTE: P.h.D. Simona Di Nino

DURATA: 8 ore

ABSTRACT: The course introduces the basics of the perturbation analysis for weakly nonlinear dynamical systems, with special reference to the multiple scale method for ordinary differential systems. The following topics are addressed: eigenvalue and eigenvector sensitivity analysis; initial value problems: straightforward expansions; the multiple scale method: basic aspects and advanced topics; Duffing oscillator under external excitation: primary, super-harmonic and sub-harmonic resonances; Duffing oscillator under parametric excitation; multi-d.o.f. quasi-Hamiltonian systems under external/parametric/internal resonances.