PHD COURSES MATHEMATICS AND MODELING 2022/23 – Schedule

ALL THE LECTURES WILL BE HELD IN THE CLASSROOM 1.1 IN THE BUILDING COPPITO 1.

All the lectures are in slots 9-11; 11-13; 15-17; apart from a few that appear in **bold**.

The remaining non-mandatory courses will take place after February 24; their schedule will be communicated later on.

- Week 16-20 january

  Monday 11.00-13.00 [GH]; 15.00-17.00 [AM]

  Tuesday 9.00-11.00 [AM]; 11.00-13.00 [PM]; 15.00-17.00 [GH]

  Wednesday 9.00-11.00 [PDE]; 11.00-13.00 [QC]; 15.00-17.00 [GH]

  Thursday 9.00-11.00 [PM]; 11.00-13.00 [PDE]; 15.00-17.00 [GH]

  Friday 9.00-11.00 [PDE]

  [GH]: Geometry of hypersurfaces (Pipoli)

  [AM]: Algebraic methods for the cryptanalysis of symmetric primitives (Civino)

  [PM]: Perturbation Methods for the Stability Analysis of Dynamical Systems (Di Nino)

  [PDE]: Introduction to Nonlinear Dispersive Pdes (Forcella)

  [QC]: Introduction to quantum computing (Guidoni)

- Week 23-27 january

  Monday ~~9.00-11.00~~ [AM]; ~~11.00-13.00~~ [GH];**~~14.00-16.00~~ [QC]** ❄❄❄

  Tuesday 9.00-11.00 [OC]; 11.00-13.00 [PM]; 15.00-17.00 [GH]

  Wednesday 9.00-11.00 [AM]; 11.00-13.00 [GH]; 15.00-17.00 [OC]

  Thursday 9.00-11.00 [PM]; 11.00-13.00 [OC]; 15.00-17.00 [GH]

  Friday 9.00-11.00 [OC]; 11.00-13.00 [AM]; 15.00-17.00 [GH]

  [OC]: Introduction to Optimal Control (Palladino, Scarinci)

- Week 30 january-3 february

  Monday 9.00-11.00 [CV]; 11.00-13.00 [OC]; 15.00-17.00 [FE]

  Tuesday 9.00-11.00 [FE]; 11.00-13.00 [OS]; 15.00-17.00 [CV]

  Wednesday 9.00-11.00 [QC]; 11.00-13.00 [OC]; 15.00-17.00 [CV]

  Thursday 9.00-11.00 [CV]; 11.00-13.00 [VM]; 15.00-17.00 [FE]

  Friday 9.00-11.00 [CV]; 11.00-13.00 [VM]

  [CV]: Direct methods in Calculus of Variations (Radici)

  [FE]: Introduction to the Finite Element Method for Partial Differential Equations (D'Ambrosio, Scalone)

  [VM]: Variational methods in continuum mechanics [VM] (Ciallella, Dell'Isola)

  [OS] Operator semigroups and applications (Engel)

- Week 6-10 february

  Monday 11.00-13.00 [VM]; 15.00-17.00 [SK]

  Tuesday 9.00-11.00 [FE]; 11.00-13.00 [QC]; 15.00-17.00 [VM]

  Wednesday 9.00-11.00 [VM]; 11.00-13.00 [SK];

  Thursday 9.00-11.00 [SK]; ~~11.00-13.00~~ [FE]; 15.00-17.00 [QC]

  Friday 11.00-13.00 [SK];

  [SK]: Introduction to the Sherrington-Kirkpatrick model and the Parisi formula (Alberici)

- Week 13-17 february

  Monday **11.30-13.30** [HJ]; 15.00-17.00 [ET]

  Tuesday 9.00-11.00 [EE]; 11.00-13.00 [OS]; 15.00-17.00 [HJ]

  Wednesday 9.00-11.00 [ET]; 11.00-13.00 [HJ]; 15.00-17.00 [EE]

  Thursday 9.00-11.00 [EE]

  [HJ]: Introduction to First-Order Hamilton-Jacobi Equations and Applications To Mean Field Games (Mendico)

  [ET]: Ergodic theory for SPDEs and its applications (Carigi)

  [EE]: Mathematical models for economic equilibria (Giuli)

- Week 20-24 february

  Monday 11.00-13.00 [HJ];

  Tuesday 9.00-11.00 [HJ]; 11.00-13.00 [OS]; 15.00-17.00 [EE]

  Wednesday 9.00-11.00 [EE]; 11.00-13.00 [ET]; 15.00-17.00 [QC]

  Thursday 9.00-11.00 [FE]; 11.00-13.00 [ET]; 15.00-17.00 [HJ]

  Friday 11.00-13.00 [QC]; **14.00-16.00 [QC]**

Abstracts

[GH] Geometry of hypersurfaces, 15 hours, G. Pipoli (Mandatory)

Abstract: In this course we will discuss some classical and recent results about the geometry of hypersurfaces with prescribed mean curvature in space forms and other homogeneous manifolds. Particular attention will be given to the case of constant mean curvature hypersurfaces, including minimal hypersurfaces (i.e. those with everywhere vanishing mean curvature), and translators (i.e. hypersurfaces evolving by mean curvature flow just translating in a fixed direction).

These classes of hypersurfaces appear naturally in different contexts: for example they are the mathematical models for soap bubbles and soap films.

The techniques and the result that we are going to discuss lie between Riemanninan geometry, differential geometry and PDEs.

[AM] Algebraic methods for the cryptanalysis of symmetric primitives, 10 hours, R. Civino (Mandatory)

Abstract: Symmetric cryptographic primitives are used to keep the user data private once the two (or more) parties involved have agreed on a shared key. The design strategies used to build modern symmetric ciphers are the final product of the endless competition between designers and cryptanalysts, being the aim of the latter to recover information they should not be entitled to. After having given a short description of the main design solutions from an algebraic point of view, in this course we will discuss several techniques of cryptanalysis which make use of algebraic tools to highlight unexpected behaviour during the encryption. Such techniques include elements of the theory of finite fields, linear algebra and group theory. We will also show how designers exploit the knowledge of new attacks to build more secure ciphers introducing appropriate countermeasures.

[PM] Perturbation Methods for the Stability Analysis of Dynamical Systems, 8 hours, S. Di Nino (Mandatory)

Abstract: The course introduces the basics of the perturbation analysis for weakly nonlinear dynamical systems, with special reference to the multiple scale method for ordinary differential systems. The following topics are addressed: eigenvalue and eigenvector sensitivity analysis; initial value problems: straightforward expansions; the multiple scale method: basic aspects and advanced topics; Duffing oscillator under external excitation: primary, super-harmonic and sub-harmonic resonances; Duffing oscillator under parametric excitation; multi-d.o.f. quasi-Hamiltonian systems under external/parametric/internal resonances.

[PDE] INTRODUCTION TO NONLINEAR DISPERSIVE PDES, L. Forcella

Abstract: This introductory course in nonlinear dispersive equations will consist of two parts: part 1, for every PhD student; part 2, for students in Analysis. At the end of each section, I will give/propose exercises and home-works.

Part 1- Preliminary tools; Introduction to dispersive models; Strichartz Estimates. Part 2- From LWP to GWP and Blowing-up solutions; Scattering; The concentration/compactness scheme - energy subcritical case.

References: [1] T. Cazenave, Semilinear Schrödinger equations, 2003 [2] H. Koch, D. Tataru, and M. Visan, Dispersive equations and nonlinear waves, 2014 [3] F. Linares and G. Ponce, Introduction to nonlinear dispersive equations, 2009 [4] T. Tao, Nonlinear dispersive equations, 2006

[QC] Introduction to quantum computing, 14 hours, L. Guidoni (Mandatory)

Abstract:The aim of the short course is to provide to students with background in mathematics and informatics the foundation of quantum computation. The course will consist of theoretical lectures as well as hands-on tutorial lead by the Quantum Computing experts from IBM-Italia.

Topics: General overview on quantum computation. Introduction to Quantum Mechanics and Qubits. Quantum circuits and algorithms. Single and double Qubit gates with examples. Present and future applications. Perspective of quantum computation and practical implementation of algorithms on the IBM-Q quantum computer and simulator.

[OC] Introduction to Optimal Control 12 hours, T. Scarinci and M. Palladino (Mandatory)

Abstract: The course aims at providing an overview of optimal control theory, with a special emphasis on dynamic constraint described by a differential inclusion F(t,x). This is a very general case since it covers the standard optimal control framework in which the dynamic constraint is a controlled differential equation as well as many others.

The main course topics that will be covered are: Introduction to Differential Inclusions, Filippov Existence Theorem; Necessary conditions for optimality; Invariance theory and derivation of Hamilton-Jacobi Equation.

[CV] Direct methods in Calculus of Variations, 10 hours, E. Radici

Abstract: In this course we consider mathematical models which can be regarded as a variational principles where the optimisation problem is associated to functionals in integral form. We introduce some weak notion of convexity (quasi-convexity, poly-convexity and rank-one convexity) and discuss the solvability of the optimisation problem via Direct Method of calculus of variations. The applicability of this method relies on the coercivity and the lower semicontinuity properties of the functionals. We focus on the existence of minimisers for quasi-convex functionals and examine the lower semicontinuity through the concept (of independent interest) of Young measures.

[FE] Introduction to the Finite Element Method for Partial Differential Equations, 10 hours, R. D'Ambrosio, C. Scalone (Mandatory)

Abstract: This is and introductory course on FEM discretizations to selected partial differential equations. The idea is to proceed by classes of problems, identifying the most appropriate numerical solvers. This type of approach is designed for PhD students involved in research areas where the numerical treatment of PDEs is required. Tentative list of topics:

FEM for elliptic problems.

FEM for parabolic problems.

FEM for hyperbolic problems.

1D e 2D grid generation.

Spectral methods.

Discontinuous Galerkin.

[VM] Variational methods in continuum mechanics, 10 hours, Ciallella, Dell'Isola (Mandatory)

Abstract: 1. Principle of Virtual Work as a fundamental postulate for mechanics Second Gradient Continuum Mechanics. Hamilton Rayleigh Principle for dissipative systems 2. Generalisation of the concept of Deformation and Stress: Necessary strong form for Equilibrium Conditions Essential and Natural Boundary Conditions 3. Piola Transformations and contact interactions for Second Gradient Continua 4. Edge and Surface contact interactions in second gradient continua: forces and double forces. Representation of contact interactions in terms of stresses, double stresses and shape of Cauchy cuts Limitations of so called Cauchy postulate 5. Some remarks on relevant aspects of history of mechanics and in particular on the development of the concepts of force, stress and couples.

References:

Spagnuolo, Mario, Francesco dell'Isola and Antonio Cazzani. "The study of the genesis of novel mathematical and mechanical theories provides an inspiration for future original research." Evaluation of Scientific Sources in Mechanics. Springer, Cham, 2022. 1-73.

Eugster, S. R., Dell'Isola, F., Fedele, R., and Seppecher, P. (2021). Piola Transformations in Second Gradient Continua.

Auffray, N., dell'Isola, F., Eremeyev, V. A., Madeo, A., and Rosi, G. (2015). Analytical continuum mechanics à la Hamilton-Piola least action principle for second gradient continua and capillary fluids. Mathematics and Mechanics of Solids, 20(4), 375-417.

dell'Isola, F., Seppecher, P., Placidi, L., Barchiesi, E., and Misra, A. (2020). Least action and virtual work principles for the formulation of generalized continuum models. Discrete and Continuum Models for Complex Metamaterials, 327.

[OS] Operator semigroups and applications, 6 hours, K. Engel (Mandatory)

Abstract: The theory of one-parameter semigroups of bounded linear operators on a Banach space provides a powerful tool to study, in a systematic and unified way, the well-posedness of a wide range of linear evolution equations. Moreover, it allows to obtain detailed information about the qualitative properties of the solutions like long-time behavior or positivity.

The aim of this short course, consisting of 3 lectures, is to give a brief introduction into this subject and to sketch some application. To this end the first part recalls the necessary notions from functional analysis and operator theory. In the second part the basic results of the abstract theory are presented and illustrated by some standard examples. Finally, these abstract results are applied to study the well-posedness and the qualitative properties of the solutions of some concrete evolution equation.

References:

[1] Engel, Nagel. One-Parameter Semigroups for Linear Evolution Equations, Graduate Texts in Mathematics, vol. 194. Springer-Verlag, 2000.

[2] Engel, Nagel. A Short Course on Operator Semigroups, Universitext. Springer-Verlag, 2006.

[SK] Introduction to the Sherrington-Kirkpatrick model and the Parisi formula, 8 hours, D. Alberici (Mandatory)

Abstract: We briefly introduce Statistical Mechanics at equilibrium: the Gibbs measure, the free energy generator functional, the thermodynamic limit and the Curie-Weiss (CW) model as an example of an exactly solvable model. We see that here the ferromagnetic interaction between the spins generates a phase transition.

We then move on to the Sherrington-Kirkpatrick (SK) model, a spin glass model studied for almost 50 years. Here the Gaussian interactions between the spins produce a frustrated system, which can have infinite distinct configurations. The solution was proposed by the nobel prize winner Giorgio Parisi, imagining that typical configurations are organized in an ultra-metric space. It took decades of work to prove the correctness of Parisi's formula with mathematical rigor.

We introduce the Parisi formula and focus on Francesco Guerra's interpolation method, with which we show that it is a lower bound for the generator functional of the SK model. The proof of the opposite inequality (which is beyond the possibilities of the course) is due to Michel Talagrand and, subsequently, Dmitry Panchenko.

[HJ] INTRODUCTION TO FIRST-ORDER HAMILTON-JACOBI EQUATIONS AND APPLICATIONS TO MEAN FIELD GAMES, 12 hours, C. Mendico

Abstract: Hamilton-Jacobi equations are nonlinear first order equations which have been first introduced in classical mechanics, but find application in many other fields of mathematics.

Our interest in these equations lies mainly in the connection with calculus of variations, optimal control and its application to Mean Field Games (MFG).

The theory of MFG has been developed in the last two decades by economists, engineers, and mathematicians in order to study decision making in very large populations of "small" interacting agents. The approach by Lasry and Lions leads to a system of nonlinear partial differential equations, the solution of which can be used to approximate the limit of an N-player Nash equilibrium as N tends to infinity. This course will be mainly focused on deterministic models in Euclidean space. We will introduce, first, the value function and the notion of viscosity solutions to H-J equations. Then, we will concentrate on the regularity of such a function as semiconcavity and Lipschitz continuity. Then, moving to MFG we have that these problems are associated with a first order pde system coupling a Hamilton-Jacobi equation, depending on the distribution of players, with a continuity equation driving such a distribution by the optimal feedback provided by the first equation. We will first prove the existence of solutions to the MFG system by a fixed point argument. Then, we will discuss uniqueness under some monotonicity condition for the coupling functions. Finally, we will study the long time behaviour of solutions following the approach of weak KAM theory.

[ET] Ergodic theory for SPDEs and its applications, 8 hours, G. Carigi (Mandatory)

Abstract: In this course we provide an overview over the theory of stochastic PDEs as SDEs on Hilbert spaces and ergodic properties of the associated dynamical systems. This abstract context is particularly relevant in the applications when we want to investigate the long-time average behaviour of physical systems. Among others, we will look at examples from geophysical fluid dynamics and the relevance of concepts from ergodic theory in the study of the climate.

[EE] Mathematical models for economic equilibria, 10 hours, M. Giuli (Mandatory)

Abstract: In science the term "equilibrium" has been widely used in physics, chemistry, biology, engineering and economics, among others, within different frameworks.

It generally refers to conditions or states of a system in which all competing inuences are balanced.

For instance, the economic equilibrium which studies the dynamics of supply, demand, and prices in an economy within several markets, can be modeled as a variational inequality problem.

In non-cooperative game involving two or more players, Nash proposed an equilibrium solution in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy.

This problem can be reformulated as a fixed point problem.

These mathematical models share an underlying common structure that allows to conveniently formulate them in a unique format of equilibrium.

The course is devoted to describe this format and it focuses on the main mathematical tools which are crucial for studying the existence and the stability of the solutions.

[CQE] Classic and quantum entropy 15 hours, D. Gabrielli

Abstract: Entropy is a key notion that is common to several research areas like, thermodynamics, statistical mechanics, communication theory, quantum physics. The main goal of this course is to give an introduction to the notion of entropy in the different frameworks and using specific problems and applications. In particular we will start recalling briefly the thermodynamic entropy giving then its statistical interpretation due to Boltzmann. After this we will show how entropy play a crucial role in probability theory and statistics and statistical mechanics. In particular we will discuss its key role in the theory of large deviations and in the study of large complex systems. We will then discuss as the same mathematical object is relevant in communication theory and will discuss the Shannon theorems. Finally we will discuss the extension of the entropy to the quantum setting.

[IP] Inverse Problems in Signal Processing: new results and open challenges, 12 hours, A. Cicone

Abstract: One classical problem in many applied fields of research, like Geophysics, Medicine, Engineering, Economy and Finance, is, given a signal, how to extract hidden information and features contained in it, like, for instance, quasiperiodicities and trends. Standard methods like Fourier and Wavelet Transform, historically used in signal processing, proved to be limited when nonlinear and nonstationary phenomena are present. For this reason in the last two decades several new nonlinear methods have been developed by many research groups around the world and they have been used extensively in the applications. In this course we will review the Empirical Mode Decomposition method and derived techniques, and introduce the Iterative Filtering technique and its generalizations. We will discuss their theoretical and numerical properties, show their limitations and discuss open problems. During the course some applications will be presented.

[MI] Metastability for the Ising model 6 hours, E. Cirillo [Uniroma1]

Abstract: The course will be organized in three lectures of two hours and will deal with the problem of the metastable behavior of lattice spin systems. After a brief introduction to the problem of metastability and to phase transitions in spin systems, the metastable behavior of the Ising model will be discussed in detail

[CA] A geometric perspective on cluster algebras, 15 hours, S. Stella

Abstract: Cluster algebras were introduced at the beginning of the century by Fomin and Zelevinsky as a tool in their study of total positivity ad Lusztig's dual canonical bases. Since then they have found applications in a very diverse spectrum of topics in mathematics. The ever expanding list of applications includes, but is in no way limited to, integrable systems, 3-dimensional topological quantum field theory, knot theory, supersymmetric gauge theories, 2d Liouville quantum gravity, string theory, quantum groups, categorification, mirror symmetry, hypergeometric integrals, and Macdonald theory. A key feature of cluster algebra is that they deeply rooted in some fundamental geometric constructions. The aim of this course, after having introduced a minimum of language, is to explore two of the most relevant one. In particular, depending on the audience, we will discuss how cluster algebras play a role in Teichmüller theory and in Poisson geometry.