PhD COURSES 2024/25

1) Variational methods in continuum mechanics

Alessandro Ciallella e Francesco dell'Isola, 10 hours

Abstract: 1. Principle of Virtual Work as a fundamental postulate for mechanics Second Gradient Continuum Mechanics. Hamilton Rayleigh Principle for dissipative systems

2. Generalisation of the concept of Deformation and Stress: Necessary strong form for Equilibrium Conditions Essential and Natural Boundary Conditions

3. Piola Transformations and contact interactions for Second Gradient Continua

4. Edge and Surface contact interactions in second gradient continua: forces and double forces. Representation of contact interactions in terms of stresses, double stresses and shape of Cauchy cuts Limitations of so called Cauchy postulate

5. Some remarks on relevant aspects of history of mechanics and in particular on the development of the concepts of force, stress and couples.

References:

Spagnuolo, Mario, Francesco dell'Isola and Antonio Cazzani. "The study of the genesis of novel mathematical and mechanical theories provides an inspiration for future original research." Evaluation of Scientific Sources in Mechanics. Springer, Cham, 2022. 1-73.

Eugster, S. R., Dell'Isola, F., Fedele, R., and Seppecher, P. (2021). Piola Transformations in Second Gradient Continua.

Auffray, N., dell'Isola, F., Eremeyev, V. A., Madeo, A., and Rosi, G. (2015). Analytical continuum mechanics à la Hamilton–Piola least action principle for second gradient continua and capillary fluids. Mathematics and Mechanics of Solids, 20(4), 375-417.

dell'Isola, F., Seppecher, P., Placidi, L., Barchiesi, E., and Misra, A. (2020). Least action and virtual work principles for the formulation of generalized continuum models. Discrete and Continuum Models for Complex Metamaterials, 327.

2) Statistical mechanics of dimers

D. Gabrielli 8 hours

Abstract: The course will be an introduction to the theory of random perfect matchings of graphs, considering in particular the cases of planar bipartite graphs. In this case there is a natural interpretation in terms of interfaces and the number of dimer covering can be computed using the Kasteleyn-Temperley-Fisher technique. This allows in some cases to compute the partition function associate to a Boltzmann measure and to study the statistical mechanics of the model.

3) Introduction to brace theory

Riccardo Aragona e Roberto Civino, 10 hours

Abstract: Braces were introduced by Rump with the aim of using ring-theoretical and group-theoretical techniques for studying non-degenerate involutive set-theoretical solutions of the Yang-Baxter equation of Mathematical Physics. In this course, we will introduce to the theory of braces, giving different approaches for studying these algebraic structure such as radical rings and regular subgroups of the holomorph group. In the first part, we will present the basic properties and results about braces and skew braces. In the second part, we will present an interesting application of braces defined over the finite field of order 2 to the cryptanalysis of block ciphers.

4) Perturbation Methods for the Stability Analysis of Dynamical Systems

Simona di Nino, 10 hours

Abstract: The course introduces the basics of the perturbation analysis for weakly nonlinear dynamical systems, with special reference to the multiple scale method for ordinary differential systems. The following topics are addressed: eigenvalue and eigenvector sensitivity analysis; initial value problems: straightforward expansions; the multiple scale method: basic aspects and advanced topics; Duffing oscillator under external excitation: primary, super-harmonic and sub-harmonic resonances; Duffing oscillator under parametric excitation; multi-d.o.f. quasi-Hamiltonian systems under external/parametric/internal resonances.

5) Conservation laws and traffic flow models

Felisia Chiarello 10 hours

Abstarct: This course will deal with nonlinear conservation laws in one space dimension and their application to traffic flow. In the first part of the course, we will show the basic theory: relationship with the Hamilton-Jacobi equation, weak solutions, admissibility conditions and shocks, solution to the Riemann problem, and the construction of a nonlinear contractive semigroup of solutions in L^1. Moreover, we will analyze weak solutions to the Cauchy problem for scalar conservation laws with initial data having small total variation. The second part of the course will concern about traffic models based on conservation laws. In particular, we will apply the theory to the study of the Lighthill-Whitham Richards (LWR) model, focusing on the relation with the microscopic Follow-the Leader (FtL) model. After that, we will generalize the LWR to the nonlocal setting.

6) Methods and algorithms of numerical optimization

Vladimir Protasov 10 hours

Abstract: This is an introductory course on modern numerical optimization, in which both the algorithmic issue and the theoretical background are analyzed. Among the main topics are:

1) Smooth optimization and its numerical site (the Lagrange multiplier rule for constrained problems, the gradient method, the simple iteration method, and the Newton method).

2) Non-smooth convex optimization. Calculus of subdifferentials and the KKT theorem. The black-box problems: Fibonacci method and cutting-plane schemes, randomization and hit-and-run algorithm, the ellipsoid method. The structural optimization: the interior point method, the semidefinite programming.

3) Numerical methods in calculus of variations and optimal control.

4) Elements of global optimization

5) Applications

7) Introduction to Optimal Transport and Applications

Emanuela Radici e Antonio Esposito 16 hours

Abstract: The theory of optimal transport began in the eighteenth century with the Monge problem (1781), which is to minimize the cost of transporting an amount of material from the given source set of origins to a target set of destinations. In the forties, Kantorovitch gave an important reformulation of the problem and, since then, the Monge-Kantorovitch problem has been a classical subject in probability theory, economics and optimization. We will discuss the existence and the properties of the optimal plan in Monge and Kantorovitch's problems under different conditions on the cost. We will exploit the relation with Kantorovitch's duality theorem, with Brenier's polar decomposition theorem, and with the Monge-Ampere equation, a PDE which arises naturally in this context. The last part of the course will be devoted to some applications of optimal transport.

8) Introduction to Optimal Control

Michele Palladino 16 hours

Abstract: The course aims at providing an overview of optimal control theory, with a special emphasis on dynamic constraint described by a differential inclusion $F(t, x)$. This is a very general case since it covers the standard optimal control framework in which the dynamic constraint is a controlled differential equation as well as many others.

The main course topics that will be covered are: Introduction to Differential Inclusions, Filippov Existence Theorem; Necessary conditions for optimality; Invariance theory and derivation of Hamilton-Jacobi Equation.

9) Introduction to quantum computing

Leonardo Guidoni 14 hours

Abstract: The present short course is a joint Ph.D. course between the Ph.D. in Mathematics and Models and the Ph.D. in Informatics. The aim of the short course is to provide to students with background in mathematics and informatics the foundation of quantum computation. The course will consist of theoretical lectures as well as hands-on tutorial lead by the Quantum Computing experts from IBM-Italia. Topics: General overview on quantum computation. Introduction to Quantum Mechanics and Qubits. Quantum circuits and algorithms.

Single and double Qubit gates with examples. Present and future applications. Perspective of quantum computation and practical implementation of algorithms on the IBM-Q quantum computer and simulator.

10) Mathematical models for economic equilibria

Massimiliano Giuli 10 hours

Abstract: In science the term equilibrium has been widely used in physics, chemistry, biology, engineering and economics, among others, within different frameworks. It generally refers to conditions or states of a system in which all competing inuences are balanced. For instance, the economic equilibrium which studies the dynamics of supply, demand, and prices in an economy within several markets, can be modeled as a variational inequality problem. In non-cooperative game involving two or more players, Nash proposed an equilibrium solution in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy. This problem can be reformulated as a fixed point problem. These mathematical models share an underlying common structure that allows to conveniently formulate them in a unique format of equilibrium. The course is devoted to describe this format and it focuses on the main mathematical tools which are crucial for studying the existence and the stability of the solutions.

11) Introduction to symplectic geometry, Hamiltonian actions and Lagrangian fibrations

Lucio Bedulli 10 hours

Abstract: The aim of this short course is to give an introduction to symplectic geometry and topology with a look to Lagrangian fibrations and their role in Mirror Symmetry.

After introducing basics on symplectic manifolds we will focus on Hamiltonian actions and moment maps.

We will then specialize to torus actions and integrable systems: here we will introduce Lagrangian fibrations and discuss Arnold-Liouville's theorem. We will emphasize the role of affine structures.

Finally we will investigate the relationship between symplectic geometry and complex geometry and see how (special) Lagrangian fibrations enter the picture of mirror symmetry.

12) Mathematics for Signal processing

Antonio Cicone 16 hours

Abstract: One classical problem in many applied fields of research, like Geophysics, Medicine, Engineering, Economy, and Finance, is, given a signal, how to extract hidden information and features contained in it, like, for instance, quasi-periodicities and trends.

Standard methods like Fourier and Wavelet Transform, historically used in signal processing, proved to be limited when nonlinear and nonstationary phenomena are present. For this reason in the last two decades, several new nonlinear methods have been developed by many research groups around the world and they have been used extensively in the applications.

In this course, we will review the Empirical Mode Decomposition method and derived techniques, and introduce the Iterative Filtering technique and its generalizations. We will discuss their theoretical and numerical properties, show their limitations, and discuss open problems.

During the course, some applications will be presented as well as an introduction to the Matlab codes available online.

13) Dynamical low rank approximation: eigenvalues, PDEs and optimisation

Carmela Scalone 15 hours

Abstract:The efficient computation of low rank solutions of matrix differential equations is a powerful tool in Numerical Analysis nowadays involved in the solution of many problems of wide interest. We start form the basics of low rank approximation in numerical linear algebra, in particular in the context of matrix nerness, to get to the dynamical approach. We focus on the problems of approximating the spectral abscissa of high dimensional operators and the solutions of partial differential equations, which are particularly meaningful in plasma physics. When evolutive problems are stated in terms of searching for low rank solutions of matrix differential equations, then the underlying dynamics is described according to the dynamical low rank appoximation approach. In this way, fixed a value of the rank, the solution is factorized, according to such value, and the differential equations for the factors are considered. In the considered approach, the numerical solution of such systems is computed by developing specific splitting methods, which can provide special adaptations to manage the variability of the rank or the management of meaningful quantities for the particular type of system under examination.

In summary: low rank approximation in numerical linear algebra, matrix nearness, dynamical low rank approximation for matrix ODEs, splitting integrators, low rank numerical solution of kinetic models, low rank computations/optimisation of eigenvalues of linear and PDEs operators.

14) Poisson random measures and applications Ida Minelli 10 hours

Abstract: The course is an introduction to the theory of Poisson random measures and Poisson point processes. In particular, we discuss the notions of Stochastic integrals with respect to point processes and Ito's formula. As an application, we will consider a general class of interacting particle systems and give a graphical construction of such processes in terms of stochastic integrals with respect to Poisson random measures. The construction gives a natural way to define ”couplings”, which are powerful tools to study the long time behavior of particle systems.

15) Lavrentiev phenomenon in the Calculus of Variations

Giulia Treu, 8 hours

Abstarct: It has been shown, by some very famous examples (due to Lavrentiev, Manià, Zhykov, et al.) that integral functionals of the Calculus of Variations may exhibit the following phenomenon: if we approximate a Sobolev function with a sequence in a dense subspace it may happen that the corresponding values of the functional do not approximate the value obtained on the original function.

A special and important case is the one where we deal with a Sobolev space and, as dense subspace, we consider the Lipschitz functions. In fact, the occurrence of this behavior gives rise to relevant problems when dealing with numeric simulations.

We will start describing the historical examples and we will discuss various results, up to very recent ones, providing sufficient conditions preventing the occurrence of the Lavrentiev phenomenon.

Giulia Treu: Dipartimento di Matematica, Università degli Studi di Padova, Via Trieste, 63-35121 Padova

e-mail: giulia.treu@unipd.it