

# A review of Continuum Thermodynamics

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# Summary

Thermodynamics of continua is not a simple subject. It deals with the interplay between balance laws and material characterization.

This is an attempt to collect from major articles and books (scattered over several decades) a few principles and methods and lay down a consistent formulation to be used for modelling problems from disparate fields.

This is not at all an original “theory”. It’s just a rearrangement of old ideas in a framework which should be easy to grasp and prove satisfactory to the modern mechanician or the (applied) mathematician.

Nevertheless it should be emphasized that the main features of this framework are:

- the balance equations are stated as *power balance laws* (the so-called “virtual power” formulation) without relying on any kind of energy;
- the coupling between different fields is described through an “energy imbalance” principle, based on the rate of change of a *free energy*, delivering constitutive restrictions according to the “Coleman-Noll” procedure;
- visco-elasticity, species diffusion in solids, poroelasticity and thermo-mechanics all share similar principles and methods;
- correspondingly, Fick’s law, Darcy’s law and Fourier’s law turn out to be constitutive laws derived from the general principles.







# Summary

The historical development is reflected both in the symbols used and in the main relations derived.






The pedagogical goal is to make the students able to read both past and current literature on the subject or a related one.

This review could even be used as a study-guide. To this purpose a handout version contains references to the main sources and to the appropriate expressions therein.






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-  W. Hong, X. Zhao, J. Zhou, Z. Suo, A theory of coupled diffusion and large deformation in polymeric gels, *J. Mech. Phys. Solids*, 56, 1779–1793, 2008.
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# Outline

## SPECIES DIFFUSION (Lithium-ion battery)

- Kinematics and kinetics
- Molar conservation law for lattice sites
- Species molar balance
- Molar flux per unit current area
- Molar source/sink density per unit current volume
- From molar balance to power balance
- Chemical potential
- Force power balance & Species power balance
- Power balance & Free energy imbalance
- Free energy imbalance and material characterization
- Fick's law



## THERMODYNAMICS

- Entropy flow balance
- Entropy flux per unit current area
- Entropy supply per unit current volume
- From entropy flow balance to power balance
- Absolute temperature
- Force power balance & Entropy flow power balance
- Internal entropy balance and Clausius-Duhem inequality
- Power balance & Free energy imbalance
- Helmholtz free energy
- Free energy imbalance and dissipation
- Fourier's law