Proposal for a short lecture within
the PhD Program Università dell'Aquila, 2019:

Control Theory and Hyperbolic Partial Differential Equations

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The aim of this course is to offer a basic training in **nonlinear hyperbolic equations** of **Hamilton-Jacobi** kind, highlighting the connection of this part of theory with the theory of optimal control. The idea is to provide the students with the basic tools necessary for the comprehension of some actual subjects of research in applied mathematics.

Entry requirements/Target: The course is intended for PhD students in Mathematics. Students are expected to have a basic knowledge of dynamic systems and linear partial differential equations. The short numerical tutorial will be supplied using the programming language MATLAB. Some basics in conservation laws, optimization or optimal control are useful but not mandatory for the full comprehension of the more advanced tasks.
If requested, the lectures may be held in English.

Structure of the lecture: The course is composed of 8 hours on 2 weeks and a tutorial of 2 hours in laboratory to be held on January-February 2019.

Reference books: Lecture notes of the course are provided to the students. [1, 3] constitute the main sources of in-dept study, secondary the more generic [2, 4].

Themes: The first 4h of the course is devoted to an operative-oriented exposition of the theory of the **Hamilton-Jacobi equations** (characterization of weak solutions, fundamental hypotheses that guarantee well-posedness of the problem), its derivation (relation with conservation laws, Dynamic programming principle, relation with optimal control problems). The other 4 hours are devoted to the introduction of numerical techniques for the approximation of the solution of these equations (Finite Differences, Semilagrangian methods) deepening in some more **advanced issues** of the matter as numerical tools for a fast resolution (Policy iteration algorithm, Fast Marching, Fast Sweeping), some non trivial extensions of the theory (Obstacle problems, Target problems, Differential games).
In both the parts, a special attention is dedicated to the discussion of some **applications** among image processing, social and biological models, financial mathematics, control systems.

References

[1] Bardi, Martino, and Italo Capuzzo-Dolcetta. Optimal control and viscosity solutions of Hamilton-Jacobi-Bellman equations. Springer Science & Business Media, 2008.

[2] Evans, Lawrence C., Partial differential equations. Graduate Studies in Math., AMS, 1994.

[3] Falcone, Maurizio, and Roberto Ferretti, Semi-Lagrangian Approximation Schemes for Linear and Hamilton-Jacobi Equations. Society for Industrial and Applied Mathematics, 2013.

[4] Quarteroni, Alfio, and Alberto Valli. Numerical approximation of partial differential equations. Vol. 23. Springer Science & Business Media, 2008.