Title: Variational derivation of continuum mechanics equations

Professor: Francesco dell'Isola and/or Luca Placidi

Aim: The aim of this short course is to introduce the students to the variational methods that are used in continuum mechanics. Static and dynamic cases will be analyzed. The mathematical derivations will be done on the basis of assumptions that are formulated on the form of the action functional. In the static case, the action is reduced to the total energy functional and a method to derive its form from a discrete model is also sketched.

Duration: 10 hours divided into 5 lectures

Arguments: The subjects developed during the course will be a suitable selection of the following ones: 3D continuum elasto-dynamic model derivations for the standard and for the strain gradient case. Euler-Bernoulli beam theory derived from an Action principle as well as from a discrete model. Continuum equations for materials with band gap for the 3D and for the 1D cases. The choice may depend on the interests of the students

MODELS: Standard and strain gradient elastic materials. Euler beams. Materials with band gap.

METHODS: Variational methods. Heuristic homogenization procedure.

Program: A possible effective program may be the following:

LECTURE 1: It is shown in details the case of 3D continuum elasto-dynamic case for the standard.
LECTURE 2: It is shown in details the case of 3D continuum elasto-dynamic case for the strain gradient case.
LECTURE 3: Standard Euler-Bernoulli beam theory will be derived from an action.
LECTURE 4: Standard Euler-Bernoulli beam theory will be derived from a discrete model.
LECTURE 5: The derivation of the continuum equations for materials with band gap for the 3D and for the 1D cases will be shown.

Bibliography: Some basic reference texts are the following:

dell'Isola Francesco, PLACIDI L (2011). Variational principles are a powerful tool also for formulating field theories. In: FRANCESCO DELL'ISOLA, SERGEY GAVRILYUK. (a cura di): FRANCESCO DELL'ISOLA, SERGEY GAVRILYUK, CISM Courses and Lectures. Variational Models and Methods in Solid and Fluid Mechanics. CISM INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES, vol. 535, p. 1-16, Wien, New York:SpringerWienNewYork, ISBN: 978-3-7091-0982-3, ISSN: 0254-1971

N. Auffray, F. dell'Isola, V. Eremeyev, A. Madeo, PLACIDI L, G. Rosi (2014). Least action principle for second gradient continua and capillary fluids: a Lagrangian approach following Piola's point of view. In: (a cura di): U. Andreaus, F. dell'Isola, R. Esposito, S. Forest, G. Maier, and U. Perego, The complete works of Gabrio Piola, volume I. Advanced Structured Materials. ADVANCED STRUCTURED MATERIALS, vol. 38, Springer Verlag, ISSN: 1869-8433, doi: 10.1007/978-3-319-00263-7_4

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Turco E., Golaszewski M., Giorgio I., Placidi L (2017). Can a hencky-type model predict the mechanical behaviour of pantographic lattices?. In: (a cura di): Sofonea M., dell'Isola F., Steigmann D., Mathematical Modelling in Solid Mechanics. ADVANCED STRUCTURED MATERIALS, vol. 69, p. 285-311, Singapore:Springer Nature, ISBN: 978-981103763-4, ISSN: 1869-8433, doi: 10.1007/978-981-10-3764-1_18