THE HILBERT CURVE OF A POLARIZED MANIFOLD: A NEW ENTRY IN ADJUNCTION THEORY?

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Abstract

Given a polarized manifold (X, L), consider the polynomial $p(x, y) := \chi(\mathcal{O}_X(xK_X + yL))$. The Hilbert curve of (X, L) is the complex affine plane algebraic curve $\Gamma := \Gamma_{(X, L)}$ defined by p(x, y) = 0, when we look at x and y as complex variables. A natural expectation is that properties of (X, L) are encoded by its Hilbert curve. In fact, Γ is sensitive with respect to fibrations on X induced by a suitable adjoint linear system to the polarizing line bundle L. In particular, if X is a projective bundle over a smooth curve it turns out that Γ has the shape of a comb, and, conjecturally, this special shape characterizes the structure of (X, L). I will give a sketch of the progress on this conjecture (joint result with Andrea L. Tironi) and discuss further interesting properties of Γ for some special varieties arising in adjunction theory.