



UNIVERSITÀ DEGLI STUDI DELL' AQUILA  
**Algorithms for Distributed Systems: Mid-term Evaluation**  
 Wednesday, November 27th, 2013 – Prof. Guido Proietti

Write your data =>	Last name: .....	First name: .....	ID number: .....	Points
EXERCISE 1				
EXERCISE 2				
EXERCISE 3				
TOTAL				

**EXERCISE 1: Multiple-choice questions (10 points)**

**Remark:** Only one choice is correct. Use the enclosed grid to select your choice. A correct answer will make you gain 3 points, while a wrong answer will charge you with a -1 penalization. The final result will be given by summing up all the obtained points (0 for a missing answer), by normalizing on a 10 base.

1. Let  $f(n)$  and  $g(n)$  denote the message complexity of the *Chang & Roberts* algorithm in the best and in the worst case, respectively. Which of the following asymptotic relations is wrong?  
 \*a)  $f(n) = \Theta(g(n) \cdot n)$    \*b)  $f(n) = \Theta(g(n))$    c)  $f(n) = o(g(n))$    d)  $f(n) = O(g(n))$
2. Let  $f(n)$  and  $g(n)$  denote the message complexity of the *Chang & Roberts* and the *Hirshberger & Sinclair* algorithm in the average and in the worst case, respectively. Which of the following asymptotic relations is correct?  
 a)  $f(n) = \Theta(g(n) \cdot \log n)$    \*b)  $f(n) = \Theta(g(n))$    c)  $f(n) = o(g(n))$    d)  $f(n) = \omega(g(n))$
3. Let us consider the *leader election* algorithm we studied for a synchronous ring with  $n$  processors, non-anonymous and uniform. Let the minimum id in the ring be equal to  $2^n$ . Then, the algorithm has a number of rounds of:  
 a)  $\Theta(2^n)$    b)  $O(1)$    \*c)  $\Theta(n \cdot 2^n)$    d)  $\Theta(n)$
4. Let us consider the first round of the synchronous version of the *Prim* algorithm on  $n$  processors. Then, in the worst case, how many messages will be sent in such a round?  
 a)  $n$    b)  $n - 1$    \*c)  $O(1)$    d) no message is sent
5. Let us consider the first phase of the synchronous version of the *GHS* algorithm on  $n$  processors. Then, in the worst case, how many rounds the phase consists of?  
 a) exactly  $n$    b)  $O(1)$    \*c)  $\Theta(n)$    d)  $O(\log n)$
6. Let  $f(n)$  and  $g(n)$  denote the message complexity of the asynchronous version of the *Prim* and the *GHS* algorithm, respectively, when executed on a sparse graph, i.e., with  $m = \Theta(n)$ . Which of the following asymptotic relations is correct?  
 a)  $f(n) = \Theta(g(n) \cdot n)$    b)  $f(n) = \Theta(g(n))$    c)  $f(n) = o(g(n))$    \*d)  $f(n) = \omega(g(n))$
7. The randomized algorithm for finding a *maximal independent set* running on a graph with  $n$  nodes and with degree  $d = O(1)$ , with high probability ends within a number of rounds in the order of:  
 \*a)  $O(\log n)$    b)  $O(1)$    c)  $O(n)$    d)  $O(n \log n)$
8. Let be given a synchronous  $n$ -processor system, with at most 1 crash failure. Assume that all the processors have a different input. Then, how many messages are totally sent during the execution of the 1-resilient to crash failures consensus algorithm?  
 \*a)  $\Theta(n^3)$    b)  $\Theta(n^2)$    c)  $\Theta(n)$    d)  $\Theta(1)$
9. Let be given a synchronous  $n$ -processor system, with at most 1 Byzantine failure. Assume that all the processors have a same input. Then, how many messages are totally sent during the execution of the 1-resilient King's algorithm?  
 a)  $\Theta(n^3)$    \*b)  $\Theta(n^2)$    c)  $\Theta(n)$    d)  $\Theta(1)$
10. In the *exponential-tree* algorithm with  $n$  processors, how many messages are sent during round  $d$ ?  
 a)  $O(n)$    b)  $O(n^2)$    c)  $O(n^d)$    \*d)  $O(n^{d+1})$

**Answer Grid**

	Question									
Choice	1	2	3	4	5	6	7	8	9	10
a										
b										
c										
d										

**EXERCISE 2: Open questions (10 points)**

**Remark:** Select at your choice one out of the following two questions, and address it exhaustively.

1. Describe and analyze the *Chang & Roberts* algorithm.
2. Prove that there is no 1-resilient to Byzantine failures algorithm for 3 processors.

**EXERCISE 3: Algorithm (10 points: 5 for the correctness, 3 for the efficiency, and 2 for the analysis)**

Design an algorithm for a non-uniform, non-anonymous, synchronous MPS  $G = (V, E)$ , with synchronous start, which computes for each processor the length of the longest cycle in  $G$  the processor belongs to (assume that rounds are of *send-recv-compute* type).