



UNIVERSITÀ DEGLI STUDI DELL'AQUILA  
**Non-Cooperative Networks: Mid-term Evaluation**  
 Tuesday, November 8th, 2016 – Prof. Guido Proietti

Write your data ⇒	Last name: .....	First name: .....	ID number: .....	Points
EXERCISE 1				
EXERCISE 2				
TOTAL				

**EXERCISE 1: Multiple-choice questions (20 points)**

**Remark:** Only one choice is correct. Use the enclosed grid to select your choice. A correct answer scores 3 points, while a wrong answer receives a  $-1$  penalization. The final score will be given by summing up all the obtained points (0 for a missing answer), by normalizing on a 20 base.

1. A *Dominant Strategy Equilibrium* is a strategy combination  $s^* = (s_1^*, \dots, s_N^*)$ , such that (assume  $p_i$  is a cost):
  - a) there exists a player  $i$  and an alternative strategy profile  $s = (s_1, \dots, s_i, \dots, s_N)$ , such that  $p_i(s_1, \dots, s_i^*, \dots, s_N) \geq p_i(s_1, \dots, s_i, \dots, s_N)$
  - \*b) for each player  $i$  and for any possible alternative strategy profile  $s = (s_1, \dots, s_i, \dots, s_N)$ ,  $p_i(s_1, \dots, s_i^*, \dots, s_N) \leq p_i(s_1, \dots, s_i, \dots, s_N)$
  - c) there exist no player  $i$  and no alternative strategy profile  $s = (s_1, \dots, s_i, \dots, s_N)$ , such that  $p_i(s_1, \dots, s_i^*, \dots, s_N) \leq p_i(s_1, \dots, s_i, \dots, s_N)$
  - d) for each player  $i$  and for any possible alternative strategy profile  $s = (s_1, \dots, s_i, \dots, s_N)$ ,  $p_i(s_1^*, \dots, s_i^*, \dots, s_N^*) \geq p_i(s_1, \dots, s_i, \dots, s_N)$
2. A *Nash Equilibrium* is a strategy combination  $s^* = (s_1^*, \dots, s_N^*)$ , such that (assume  $p_i$  is a utility):
  - a) there exists a player  $i$  and an alternative strategy profile  $s = (s_1, \dots, s_i, \dots, s_N)$ , such that  $p_i(s_1, \dots, s_i^*, \dots, s_N) \leq p_i(s_1, \dots, s_i, \dots, s_N)$
  - b) for each player  $i$  and for any possible alternative strategy profile  $s = (s_1, \dots, s_i, \dots, s_N)$ ,  $p_i(s_1, \dots, s_i^*, \dots, s_N) \geq p_i(s_1, \dots, s_i, \dots, s_N)$
  - c) there exist no player  $i$  and no alternative strategy profile  $s = (s_1, \dots, s_i, \dots, s_N)$ , such that  $p_i(s_1, \dots, s_i^*, \dots, s_N) \leq p_i(s_1, \dots, s_i, \dots, s_N)$
  - \*d) for each player  $i$  and for any alternative strategy  $s_i$  of  $i$ ,  $p_i(s_1^*, \dots, s_i^*, \dots, s_N^*) \geq p_i(s_1^*, \dots, s_i, \dots, s_N^*)$
3. How the Price of Anarchy is defined for a game in which the social choice function  $C$  has to be minimized ( $S$  is the set of Nash equilibria)?
  - \*a)  $\text{PoA} = \sup_{s \in S} \frac{C(s)}{C(\text{OPT})}$
  - b)  $\text{PoA} = \inf_{s \in S} \frac{C(s)}{C(\text{OPT})}$
  - c)  $\text{PoA} = \sup_{s \in S} \frac{C(\text{OPT})}{C(s)}$
  - d)  $\text{PoA} = \inf_{s \in S} \frac{C(\text{OPT})}{C(s)}$
4. How the Price of Stability is defined for a game in which the social-choice function  $C$  has to be maximized ( $S$  is the set of Nash equilibria)?
  - \*a)  $\text{PoS} = \sup_{s \in S} \frac{C(s)}{C(\text{OPT})}$
  - b)  $\text{PoS} = \inf_{s \in S} \frac{C(s)}{C(\text{OPT})}$
  - c)  $\text{PoS} = \sup_{s \in S} \frac{C(\text{OPT})}{C(s)}$
  - d)  $\text{PoS} = \inf_{s \in S} \frac{C(\text{OPT})}{C(s)}$
5. In a network with degree- $p$  polynomials latency functions,  $p > 1$ , the cost of a Nash flow is  $x$  times that of the min-cost flow, where  $x$  is:
  - a)  $O(p)$
  - b)  $O(\log p)$
  - \*c)  $O(p/\log p)$
  - d)  $4/3$
6. In the global connection game on a graph  $G = (V, E, c)$ , if we denote by  $c_e$  (resp.,  $k_e$ ) the cost (resp., the load) of an edge  $e \in E$ , and by  $N(S)$  the network induced by a given strategy profile  $S$ , which of the following is the social-choice function?
  - a)  $C(S) = \sum_{e \in N(S)} c_e \cdot H_{k_e}$
  - b)  $C(S) = \sum_{e \in N(S)} c_e/k_e$
  - \*c)  $C(S) = \sum_{e \in N(S)} c_e$
  - d)  $C(S) = \sum_{e \in E} c_e$
7. In a global connection game with  $k$  players, which of the following claim is false?
  - a) there exists an instance such that  $\text{PoS} = H_k$
  - b)  $\text{PoA} \leq k$
  - c)  $\text{PoS} \leq H_k$
  - \*d)  $\text{PoA} \leq H_k$
8. In the local connection game on a set of nodes  $V$ , if we denote by  $\alpha$  the cost of activating an edge, by  $n_u$  the number of edges bought by a player  $u \in V$ , and finally by  $\text{dist}_{G(S)}(u, v)$  the distance between  $u$  and  $v$  in the graph  $G(S)$  induced by a given strategy profile  $S$ , which of the following is the cost function for player  $u$  with respect to  $S$ ?
  - a)  $c_u(S) = \alpha \cdot \sum_{v \in V} \text{dist}_{G(S)}(u, v)$
  - \*b)  $c_u(S) = \alpha \cdot n_u + \sum_{v \in V} \text{dist}_{G(S)}(u, v)$
  - c)  $c_u(S) = \alpha + \sum_{v \in V} \text{dist}_{G(S)}(u, v)$
  - d)  $c_u(S) = \sum_{v \in V} \text{dist}_{G(S)}(u, v)$
9. In the local connection game on a set  $V$  of  $n$  nodes, if we denote by  $\alpha$  the cost of activating an edge, which of the following is a lower bound on the social-cost function of an optimal solution  $G = (V, E)$ ?
  - a)  $(\alpha - 2)n + 2n(n - 1)$
  - b)  $\alpha + 2n(n - 1)|E|$
  - c)  $(\alpha - 2)|E| + 2n^2$
  - \*d)  $(\alpha - 2)|E| + 2n(n - 1)$
10. Let be given a local connection game on a set  $V$  of  $n$  nodes, in which the cost of activating an edge is  $\alpha = 0.9$ . What is the PoA of such a game?
  - \*a) exactly 1
  - b) exactly  $6 \cdot \sqrt{0.9} + 3$
  - c) at least  $6 \cdot \sqrt{0.9} + 3$
  - d) at least 1

**Answer Grid**

Choice	Question									
	1	2	3	4	5	6	7	8	9	10
a										
b										
c										
d										

**EXERCISE 2: Open question (10 points)**

**Remark:** Select at your choice one out of the following two questions, and address it exhaustively.

1. Describe and analyze the selfish routing game.
2. Describe and analyze the local connection game.