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STABLE CONSTANT MEAN CURVATURE HYPERSURFACES

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Abstract. Let $\mathcal{N}^{n+1}$ be a Riemannian manifold with sectional curvatures uniformly bounded from below. When $n = 3, 4$, we prove that there are no complete (strongly) stable H-hypersurfaces, without boundary, provided $|H|$ is large enough. In particular, we prove that there are no complete strongly stable H-hypersurfaces in $\mathbb{R}^{n+1}$ without boundary, $H \neq 0$.

1. Introduction

Consider a Riemannian manifold $\mathcal{N}$ of dimension $n+1$ with sectional curvatures uniformly bounded from below; denote by $\sec(\mathcal{N})$ the infimum of the sectional curvatures of $\mathcal{N}$. Let M be an immersed submanifold of codimension one and let H be the mean curvature of M in the metric induced by the immersion. If H is constant, we call M an H-hypersurface. We prove the following diameter estimate.

Theorem 1. *Let $M^n \subset \mathcal{N}^{n+1}$ be a stable complete H-submanifold, $n = 3, 4$. There exists a constant $c = c(n, H, \sec(\mathcal{N}))$ such that for any $p \in M$ one has: $\mathrm{dist}_M(p, \partial M) \leq c$ whenever $|H| > 2\sqrt{|\min\{0, \sec(\mathcal{N})\}|}$.*

For the definition of stability, see Section 2. Particular cases of the previous Theorem in $\mathbb{R}^3$, $\mathbb{H}^3$, $\mathbb{H}^2 \times \mathbb{R}$ and any homogeneously regular three-manifold are proved in [9], [5], [7], [8], respectively.

We wonder if Theorem 1 holds in all dimensions.

Corollary 1. *Let M^n be a complete stable H-hypersurface of $\mathcal{N}^{n+1}$. If $n = 3, 4$ and $|H| > 2\sqrt{|\min\{0, \sec(\mathcal{N})\}|}$, then $\partial M \neq \emptyset$.*

In [12] it is proved that an H-hypersurface in $\mathbb{R}^{n+1}$, with finite total curvature, is minimal, so, if it is stable, it is a hyperplane (cf. [4]). For $n = 3, 4$, we are able to generalize this result in the following sense. We do not need the finite total curvature hypothesis on M, and the ambient space can be any manifold with uniformly bounded sectional curvature, provided the mean curvature $|H|$ is large enough (see Corollary 1).

As a consequence of the diameter estimate in Theorem 1, we have the Maximum Principle at Infinity.

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Theorem 2. *Let $\mathcal{N}^{n+1}$ have uniformly bounded sectional curvature, $n = 3, 4$. If $|H| > 2\sqrt{|\min\{0, \sec(\mathcal{N})\}|}$ and M_1, M_2 are properly embedded H-hypersurfaces in $\mathcal{N}^{n+1}$, which bound a connected domain W, then the mean curvature vector points out of W along the boundary of W.*

The proof of Theorem 2 is the same as in [8], where the result is proved for $n = 2$.

After this paper was submitted for publication, we received a preprint of Xu Cheng, where she also establishes our Theorem 1 [3].

2. Proofs

Let M be an H-hypersurface in a manifold $\mathcal{N}$ and let N be a unit vector field normal to M in $\mathcal{N}$. The stability operator of M is $L = \Delta + |A|^2 + Ric(N)$, where $Ric(N)$ is the Ricci curvature of the ambient manifold $\mathcal{N}$ in the direction of N and A is the shape operator of the immersion. We say that M is *stable* if

$$-\int_M uLu \geq 0$$

for any smooth function u with compact support on M. Our definition of stability is usually known as *strong stability*. The usual definition of stability (*weak stability*) also requires the test function u to satisfy $\int_M u = 0$. Geodesic spheres in a space form are weakly stable but they are not stable (cf. [2]). We remark that the solutions of the Plateau problem are stable hypersurfaces in our sense as well as any H-hypersurface transverse to some Killing vector field of the ambient manifold. The proof of the latter is standard (cf. for example [7]). For further relations between the two notions of stability, see [1] and [2].

Proof of Theorem 1. Consider the traceless operator $\Phi = A - HI$. One can write the stability operator of M in terms of Φ, namely, $L = \Delta + |\Phi|^2 + nH^2 + Ric(N)$. Since M is stable, there exists a function $u > 0$ on M such that $Lu = 0$ on M (cf. [6]).

Denote by ds^2 the metric on M induced by the immersion in $\mathcal{N}$ and and let $d\tilde{s}^2 = u^{2k} ds^2$, with $\frac{5(n-1)}{4n} \leq k < \frac{4}{n-1}$. This choice of k will be justified later. Notice that, in order to have some k satisfying the previous inequality, one needs $n = 3, 4$.

Consider $p \in M$ and let $r > 0$ be such that the intrinsic ball B_r of M, centered at p of ds-radius r, is contained in the interior of M. Let γ be a $d\tilde{s}$-minimizing geodesic in B_r joining p to ∂B_r. Let a be the ds-length of γ. Then $a \geq r$ and it is enough to prove that there exists a constant $c(n, H, \sec(\mathcal{N})$ such that $a \leq c$.

Let R and $\widetilde{R}$ be the curvature tensors of M in the metrics ds and $d\tilde{s}$, respectively. Choose a basis $\{\tilde{e}_1 = \frac{\partial\gamma}{\partial\tilde{s}}, \tilde{e}_2, \ldots, \tilde{e}_n\}$ orthonormal for the metric $d\tilde{s}$, such that $\tilde{e}_2, \ldots, \tilde{e}_n$ are parallel along γ and let $\tilde{e}_{n+1} = N$. The basis $\{e_1 = \frac{\partial\gamma}{\partial s} = u^k\tilde{e}_1, e_2 = u^k\tilde{e}_2, \ldots, e_n = u^k\tilde{e}_n\}$ is orthonormal for the metric ds. Denote by R_{11} and $\widetilde{R}_{11}$ the Ricci curvatures in the direction of e_1 for the metrics ds and $d\tilde{s}$, respectively. Let $\widehat{R}$ be the curvature tensor of the ambient manifold $\mathcal{N}$ and write $Ric(N) = \widehat{R}_{n+1,n+1}$.

Let $\tilde{r}$ be the length of γ in the $d\tilde{s}$ metric. Since γ is $d\tilde{s}$ minimizing, by the second variational formula, one has

$$\int_0^{\tilde{r}} \left[(n-1)\left(\frac{d\varphi}{d\tilde{s}}\right)^2 - \widetilde{R}_{11}\varphi^2\right] d\tilde{s} \geq 0, \tag{1}$$

for any smooth function φ such that $\varphi(0) = \varphi(\tilde{r}) = 0$.

As is proved in the Appendix,

$$\widetilde{R}_{11} = u^{-2k}\left\{R_{11} - k(n-2)(\ln u)_{ss} - k\frac{\Delta u}{u} + k\frac{|\nabla u|^2}{u^2}\right\}. \tag{2}$$

Now use that $Lu = (\Delta + |\Phi|^2 + nH^2 + \widehat{R}_{n+1,n+1})u = 0$ to obtain

$$\widetilde{R}_{11} = u^{-2k}\left\{R_{11} - k(n-2)(\ln u)_{ss} + k(|\Phi|^2 + nH^2 + \widehat{R}_{n+1,n+1}) + k\frac{|\nabla u|^2}{u^2}\right\}. \tag{3}$$

From the Gauss equation one has

$$R_{ijij} = \widehat{R}_{ijij} + h_{ii}h_{jj} - h_{ij}^2, \tag{4}$$

which can be rewritten as

$$R_{ijij} = \widehat{R}_{ijij} + (\Phi_{ii} + H)(\Phi_{jj} + H) - (\Phi_{ij} + H\delta_{ij})^2.$$

Taking $i = 1$ and summing up in $j = 2, \ldots, n$ we obtain

$$R_{11} = \sum_{j=2}^{n}\widehat{R}_{1j1j} + \sum_{j=2}^{n}\Phi_{11}\Phi_{jj} + (n-2)H\Phi_{11} + \sum_{j=1}^{n}\Phi_{jj}H + (n-1)H^2 - \sum_{j=2}^{n}\Phi_{1j}^2.$$

Since $\sum_{j=1}^{n}\Phi_{jj} = 0$, we have

$$R_{11} = \sum_{j=2}^{n}\widehat{R}_{1j1j} - \Phi_{11}^2 + (n-2)H\Phi_{11} + (n-1)H^2 - \sum_{j=2}^{n}\Phi_{1j}^2.$$

Replacing the last relation in equation (3) yields

$$\begin{aligned}\widetilde{R}_{11} = u^{-2k}&\Big[\sum_{j=2}^{n}\widehat{R}_{1j1j} + k\widehat{R}_{n+1,n+1} + (kn+n-1)H^2 + (n-2)H\Phi_{11}\Big]\\ &+u^{-2k}\Big[k|\Phi|^2 - \Phi_{11}^2 - \sum_{j=2}^{n}\Phi_{1j}^2 - k(n-2)(\ln u)_{ss} + k\frac{|\nabla u|^2}{u^2}\Big].\end{aligned}$$

Combining the last equation with inequality (1) gives (by abuse of notation we denote again by φ the composition $\varphi \circ \tilde{s}$; hence $\varphi(0) = \varphi(a) = 0$)

$$\begin{aligned}(n-1)\int_0^a (\varphi_s)^2 u^{-k}ds \geq \int_0^a \varphi^2 u^{-k}\Big[\sum_{j=2}^{n}\widehat{R}_{1j1j} + k\widehat{R}_{n+1,n+1}\Big]ds\\ + \int_0^a \varphi^2 u^{-k}\Big[(kn+n-1)H^2 + (n-2)H\Phi_{11} + k|\Phi|^2 - \Phi_{11}^2 - \sum_{j=2}^{n}\Phi_{1j}^2\Big]ds\\ - \int_0^a \varphi^2 u^{-k}\Big[k(n-2)(\ln u)_{ss} + k\frac{|\nabla u|^2}{u^2}\Big]ds.\end{aligned}$$

Replace φ by $\varphi u^{\frac{k}{2}}$ to get rid of u^k in the denominator. The last relation becomes

$$(n-1)\int_0^a (\varphi_s)^2 ds + k(n-1)\int_0^a \varphi\varphi_s u_s u^{-1} ds + \frac{k^2(n-1)}{4}\int_0^a \varphi^2 u_s^2 u^{-2} ds$$
$$\geq \int_0^a \varphi^2\left[\sum_{j=2}^n \widehat{R}_{1j1j} + k\widehat{R}_{n+1,n+1}\right] ds$$
$$+\int_0^a \varphi^2\left[(kn+n-1)H^2 + (n-2)H\Phi_{11} + k|\Phi|^2 - \Phi_{11}^2 - \sum_{j=2}\Phi_{1j}^2\right] ds \tag{5}$$
$$-\int_0^a \varphi^2\left[k(n-2)(\ln u)_{ss} + k\frac{|\nabla u|^2}{u^2}\right] ds.$$

Integration by parts gives

$$\int \varphi^2 (\ln u)_{ss} ds = -2\int \varphi\varphi_s \frac{u_s}{u} ds.$$

Then, replacing in inequality (5), we obtain

$$(n-1)\int_0^a (\varphi_s)^2 ds \geq k(n-3)\int_0^a \varphi\varphi_s u_s u^{-1} ds - \frac{(n-1)}{4}\int_0^a \varphi^2 (\ln u^k)_s^2 ds$$
$$+k\int_0^a \varphi^2 \frac{|\nabla u|^2}{u^2} ds + \int_0^a \varphi^2\left[k\widehat{R}_{n+1,n+1} + \sum_{j=2}^n \widehat{R}_{1j1j}\right] ds$$
$$+\int_0^a \varphi^2\left[(kn+n-1)H^2 + (n-2)H\Phi_{11} + k|\Phi|^2 - \Phi_{11}^2 - \sum_{j=2}^n \Phi_{1j}^2\right] ds,$$

that is,

(6)
$$(n-1)\int_0^a (\varphi_s)^2 ds \geq k(n-3)\int_0^a \varphi\varphi_s u_s u^{-1} ds + \left[\frac{1}{k} - \frac{(n-1)}{4}\right]\int_0^a \varphi^2 (\ln u^k)_s^2 ds$$
$$+\int_0^a \varphi^2\left[k\widehat{R}_{n+1,n+1} + \sum_{j=2}^n \widehat{R}_{1j1j}\right] ds$$
$$+\int_0^a \varphi^2\left[(kn+n-1)H^2 + (n-2)H\Phi_{11} + k|\Phi|^2 - \Phi_{11}^2 - \sum_{j=2}^n \Phi_{1j}^2\right] ds.$$

We now use that $a^2+b^2 \geq -2ab$ with $a=(n-2)H$ and $b=\frac{\Phi_{11}}{2}$ to obtain

$$(n-2)^2 H^2 + \frac{\Phi_{11}^2}{4} \geq -(n-2)H\Phi_{11}.$$

Replacing in inequality (6) yields

(7)
$$(n-1)\int_0^a (\varphi_s)^2 ds \geq k(n-3)\int_0^a \varphi\varphi_s u_s u^{-1} ds + \left[\frac{1}{k} - \frac{(n-1)}{4}\right]\int_0^a \varphi^2 (\ln u^k)_s^2 ds$$
$$+\int_0^a \varphi^2\left[k\widehat{R}_{n+1,n+1} + \sum_{j=2}^n \widehat{R}_{1j1j} + (kn-n^2+5n-5)H^2 + \right] ds$$
$$+\int_0^a \left[k|\Phi|^2 - \frac{5}{4}\Phi_{11}^2 - \sum_{j=2}^n \Phi_{1j}^2\right] ds.$$

We will now prove that the last term in inequality (7) is greater than or equal to zero. We know that

$$|\Phi|^2 \geq \Phi_{11}^2 + \Phi_{22}^2 + \cdots + \Phi_{nn}^2 + 2\sum_{j=2}^{n} \Phi_{1j}^2$$

and since $\sum_{j=1}^{n} \Phi_{jj} = 0$, we have

$$|\Phi|^2 \geq \frac{n}{n-1}\Phi_{11}^2 + 2\sum_{j=2}^{n} \Phi_{1j}^2. \tag{8}$$

Since $k \geq \frac{5(n-1)}{4n}$, using inequality (8), we obtain

$$\begin{aligned} k|\Phi|^2 &- \frac{5}{4}\Phi_{11}^2 - \sum_{j=2}^{n} \Phi_{1j}^2 \\ &\geq \frac{5}{4}\Phi_{11}^2 + \frac{5(n-1)}{2n}\sum_{j=2}^{n} \Phi_{1j}^2 - \frac{5}{4}\Phi_{11}^2 - \sum_{j=2}^{n} \Phi_{1j}^2 = \frac{3n-5}{2n}\sum_{j=2}^{n} \Phi_{1j}^2 \geq 0. \end{aligned}$$

Then, inequality (7) yields

$$\begin{aligned} (n-1)\int_0^a (\varphi_s)^2 ds &\geq (n-3)\int_0^a \varphi\varphi_s(\ln u^k)_s ds + \left[\frac{1}{k} - \frac{(n-1)}{4}\right]\int_0^a \varphi^2(\ln u^k)_s^2 ds \\ &+ \int_0^a \varphi^2\left[k\widehat{R}_{n+1,n+1} + \sum_{j=2}^{n} \widehat{R}_{1j1j} + (kn - n^2 + 5n - 5)H^2\right] ds. \end{aligned} \tag{9}$$

We now use that $a^2 + b^2 \geq -2ab$ with $a = \left(\frac{1}{k} - \frac{(n-1)}{4}\right)^{\frac{1}{2}}\varphi(\ln u^k)_s$ and $b = \frac{(n-3)}{2}\left(\frac{1}{k} - \frac{(n-1)}{4}\right)^{-\frac{1}{2}}\varphi_s$ to obtain

$$\left(\frac{1}{k} - \frac{(n-1)}{4}\right)\varphi^2(\ln u^k)_s^2 + \frac{(n-3)^2}{4}\left(\frac{1}{k} - \frac{(n-1)}{4}\right)^{-1}\varphi_s^2 \geq -(n-3)\varphi\varphi_s(\ln u^k)_s.$$

The last inequality together with inequality (9) gives

$$\begin{aligned} (n-1)\int_0^a (\varphi_s)^2 ds &\geq -\frac{(n-3)^2}{4}\left(\frac{1}{k} - \frac{(n-1)}{4}\right)^{-1}\int_0^a (\varphi_s)^2 ds \\ &+ \int_0^a \varphi^2\left[(kn - n^2 + 5n - 5)H^2 + k\widehat{R}_{n+1,n+1} + \sum_{j=2}^{n} \widehat{R}_{1j1j}\right] ds. \end{aligned}$$

Then setting $A = \frac{4[k(2-n)+(n-1)]}{4-k(n-1)}$ and making a suitable choice of a positive constant B, we can rewrite the last inequality as

$$A\int_0^a (\varphi_s)^2 ds \geq B\int_0^a \varphi^2 ds. \tag{10}$$

We remark that A is positive as soon as $k < \frac{4}{n-1} \leq \frac{n-1}{n-2}$. We now want to choose B such that

$$0 < B \leq (kn - n^2 + 5n - 5)H^2 + \left(k\widehat{R}_{n+1,n+1} + \sum_{j=2}^{n} \widehat{R}_{1j1j}\right).$$

When the curvature of the ambient manifold is nonnegative, we set $B = (kn - n^2 + 5n - 5)H^2$, which is positive if $H \neq 0$ (remember that $k > \frac{5(n-1)}{4n}$ and that $n = 3, 4$). In this case we can set $c_1 = 0$.

Otherwise, we proceed as follows. By a straightforward computation one has

$$k\widehat{R}_{n+1,n+1} + \sum_{j=2}^{n} \widehat{R}_{1j1j} \geq (kn + n - 1) \inf\{\text{sectional curvatures of } \mathcal{N}\} = \sec(\mathcal{N}).$$

Then we set $B = (kn - n^2 + 5n - 5)H^2 + (kn + n - 1)\sec(\mathcal{N})$. If

$$H^2 > \frac{kn + n - 1}{kn - n^2 + 5n - 5} \sec(\mathcal{N}), \tag{11}$$

then B is positive. In this case, one can set $c_1 = 2\sqrt{|\sec(\mathcal{N})|}$ (using the restrictions on k one can prove that $\frac{kn+n-1}{kn-n^2+5n-5} < 4$).

Integration by parts in inequality (10) yields

$$\int_0^a (\varphi_{ss} A + B\varphi)\varphi ds \leq 0.$$

Choosing $\varphi = \sin(\pi s a^{-1})$, $s \in [0, a]$ one has

$$\int_0^a \left[B - \frac{A\pi^2}{a^2}\right] \sin^2(\pi s a^{-1}) ds \leq 0.$$

Finally

$$B - \frac{A\pi^2}{a^2} \leq 0,$$

and this gives the desired inequality if we choose

$$c = \frac{2\pi\sqrt{k(2-n) + (n-1)}}{\sqrt{(4 - k(n-1))[(kn - n^2 + 5n - 5)H^2 + (kn + n - 1)\min\{0, \sec(\mathcal{N})\}]}}.$$

□

Proof of Corollary 1. Assume that such an M exists. In the proof of Theorem 1, we showed that the radius of an intrinsic disc of M that does not touch ∂M is at most c. Hence, when $\partial M = \emptyset$, the diameter of M is at most c and then M is compact. As M is stable, there exists a positive function f on M such that $L(f) = 0$ (cf. [6]). Let $p \in M$ be a minimum of the function f. At p, one has

$$0 \leq \Delta f(p) = -(|\Phi|^2(p) + nH^2 + \widehat{R}_{n+1,n+1}(p))f(p).$$

By our choice of H, the potential $|\Phi|^2 + nH^2 + \widehat{R}_{n+1,n+1}$ is strictly positive on M; hence the previous inequality yields a contradiction. □

3. Appendix

The transformation law of the curvature under the conformal change of the metric $d\tilde{s}^2 = u^{2k} ds^2$ is the following (cf. [10] page 184 and [11] formula (4)):

$$\begin{aligned} \widetilde{R}_{11} = \widetilde{Ric}(\frac{\partial\gamma}{\partial\tilde{s}}, \frac{\partial\gamma}{\partial\tilde{s}}) = \Big\{ & Ric(\frac{\partial\gamma}{\partial\tilde{s}}, \frac{\partial\gamma}{\partial\tilde{s}}) - k(n-2)\,\mathrm{Hess}(\ln u)\,(\frac{\partial\gamma}{\partial\tilde{s}}, \frac{\partial\gamma}{\partial\tilde{s}}) \\ & + k^2(n-2)\Big|\frac{\partial\gamma}{\partial\tilde{s}}(\ln u)\Big|^2 - \big[k\Delta(\ln u) + k^2(n-2)|\nabla \ln u|^2\big]u^{-2k} \Big\}. \end{aligned} \tag{12}$$

In order to simplify this equation we need to compute $\nabla_{\frac{\partial\gamma}{\partial s}} \frac{\partial\gamma}{\partial s}$.

Using the relation between the connections of conformal metrics we obtain

$$\widetilde{\nabla}_{\frac{\partial\gamma}{\partial s}}\frac{\partial\gamma}{\partial s} = \nabla_{\frac{\partial\gamma}{\partial s}}\frac{\partial\gamma}{\partial s} + 2k < \nabla \ln u, \quad \frac{\partial\gamma}{\partial s} > \frac{\partial\gamma}{\partial s} - k\nabla \ln u.$$

Since γ is geodesic in the $d\tilde{s}^2$ metric we have that $\widetilde{\nabla}_{\frac{\partial\gamma}{\partial\tilde{s}}}\frac{\partial\gamma}{\partial\tilde{s}} = 0$ and thus

$$\widetilde{\nabla}_{\frac{\partial\gamma}{\partial s}}\frac{\partial\gamma}{\partial s} = k < \nabla \ln u, \quad \frac{\partial\gamma}{\partial s} > \frac{\partial\gamma}{\partial s}.$$

The last two equations yield

$$\nabla_{\frac{\partial\gamma}{\partial s}}\frac{\partial\gamma}{\partial s} = k(\nabla \ln u)^{\perp}, \tag{13}$$

where $(\nabla \ln u)^{\perp}$ means the component of $\nabla \ln u$ perpendicular to $\frac{\partial\gamma}{\partial s}$. Now we observe that

$$\begin{aligned} \mathrm{Hess}(\ln u)\left(\frac{\partial\gamma}{\partial\tilde{s}}, \frac{\partial\gamma}{\partial\tilde{s}}\right) &= u^{-2k}\left((\ln u)_{ss} - \left(\nabla_{\frac{\partial\gamma}{\partial s}}\frac{\partial\gamma}{\partial s}\right)\ln u\right) \\ &= u^{-2k}\left((\ln u)_{ss} - k|(\nabla \ln u)^{\perp}|^2\right), \end{aligned}$$

where in the last equality we use (13). Replacing this last equation in (12) one obtains

$$\begin{aligned} \widetilde{R}_{11} &= u^{-2k}\left\{R_{11} - k(n-2)(\ln u)_{ss} + k^2(n-2)|(\nabla \ln u)^{\perp}|^2 \right. \\ &\quad \left. + k^2(n-2)|(\ln u)_s|^2 - \left[k\Delta(\ln u) + k^2(n-2)|\nabla \ln u|^2\right]\right\}, \end{aligned}$$

which can be rewritten as

$$\begin{aligned} \widetilde{R}_{11} &= u^{-2k}\left\{R_{11} - k(n-2)(\ln u)_{ss} - k\Delta(\ln u)\right\} \\ &= u^{-2k}\left\{R_{11} - k(n-2)(\ln u)_{ss} - k\frac{\Delta u}{u} + k\frac{|\nabla u|^2}{u^2}\right\}. \end{aligned} \tag{14}$$

References

[1] J.L. Barbosa, P. Bérard: *Eigenvalue and "Twisted" Eigenvalue Problems, Applications to CMC Surfaces*, J. Math. Pures Appl. 79, 5 (2000) 427-450. MR1759435 (2001f:58064)

[2] J.L. Barbosa, M. do Carmo: *Stability of Hypersurfaces with Constant Mean Curvature,* Math. Zeit. 185, 3 (1984) 339-353. MR0731682 (85k:58021c)

[3] X. Cheng: *On Constant Mean Curvature Hypersurfaces with Finite Index,* Preprint, to be published in Archiv der Mathematik.

[4] M. do Carmo, C.K. Peng: *Stable Complete Minimal Hypersurfaces,* Proceedings of the 1980 Beijing Symposium on Diff. Geom. and Diff. Eq., Science Press, Beijing, (1982) 1349–1358. MR0714373 (85e:53007)

[5] R. F. DeLima, W. Meeks, III: *Maximum Principles at Infinity for Surfaces of Bounded Mean Curvature in* $\mathbb{R}^3$ *and* $\mathbb{H}^3$, Indiana Univ. Math. Journal 53, 5 (2004) 1211–1223. MR2104275 (2005h:53011)

[6] D. Fischer-Colbrie: *On Complete Minimal Surfaces with finite Morse Index in three Manifolds,* Inv. Math. 82 (1985) 121-132. MR0808112 (87b:53090)

[7] B. Nelli, H. Rosenberg: *Global Properties of Constant Mean Curvature Surfaces in* $\mathbb{H}^2 \times \mathbb{R}$, to appear in Pacific Jour. of Math. (2004), http://www.math.jussieu.fr/∼rosen.

[8] H. Rosenberg: *Constant Mean Curvature Surfaces in Homogeneously Regular 3-Manifold,* Preprint (2005) http://www.math.jussieu.fr/∼ rosen/. MR1483380

[9] A. Ros, H. Rosenberg: *Properly Embedded Surfaces with Constant Mean Curvature,* Preprint (2001), http://www.math.jussieu.fr/∼rosen.

[10] R. Schoen, S.T. Yau: *Lectures on Differential Geometry,* International Press, (1994). MR1333601 (97d:53001)

[11] Y. Shen, S. Zhu: *Rigidity of Stable Minimal Hypersurfaces,* Math. Annalen 399 (1997) 107-116. MR1467649 (98g:53113)

[12] Y.B. Shen, X.H. Zhu: *On Complete Hypersurfaces with Constant Mean Curvature and Finite L^p-norm Curvature in* $\mathbb{R}^{n+1}$, Acta Math. Sinica, English series 21, 3 (2005) 631-642. MR2145917 (2006f:53088)

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