BULLETIN OF THE BRAZILIAN MATHEMATICAL SOCIETY

Bull Braz Math Soc, New Series 38(4), 661-664

Errata: "Minimal Surfaces in $\mathbb{H}^2 \times \mathbb{R}$", [Bull. Braz. Soc., 33 (2002), 263-292]

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Abstract. In $\mathbb{H}^2 \times \mathbb{R}$ one has catenoids, helicoids and Scherk-type surfaces. A Jenkins-Serrin type theorem holds here. Moreover there exist complete minimal graphs in $\mathbb{H}^2$ with arbitrary continuous asymptotic values. Finally, a graph on a domain of $\mathbb{H}^2$ cannot have an isolated singularity.

Keywords: minimal graph, hyperbolic plane.

Mathematical subject classification: 53A10.

1 Errata Corrige

In the following we will use the notation of the article "Minimal Surfaces in $\mathbb{H}^2 \times \mathbb{R}$".

1. Page 264 in [4]. Formula (1) should be replaced by

$$\operatorname{div}\left(\frac{\nabla u}{\tau_u}\right) = \frac{2H}{F}$$

2. Page 267. Theorem 1 in [4]. Minimal catenoids C_t in $\mathbb{H}^2 \times \mathbb{R}$ exist only for $t \in (0, \frac{\pi}{2})$ (see Proposition 5.1 in [3], Theorem 15 in [8]).

3. In the proof of Step 1 of Theorem 3 in [4], there was a mistake: page 276 lines 6-14 and Figure 7(b).

   We need to prove that the sequence $\{u_n\}$ is uniformly bounded on compact subsets of D.

   Given a complete geodesic α in $\mathbb{H}^2$, E one of the components of $\mathbb{H}^2 \setminus \alpha$, there exists a minimal graph h defined on D, asymptotic to $+\infty$ on α and

Received 28 August 2007.

to zero on $\partial_\infty(E)$ (see Figure 1 in the disk model of $\mathbb{H}^2$). This function was found independently by U. Abresch and R. Sa Earp (see [6]).

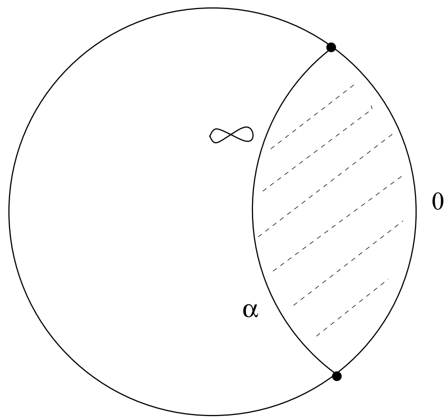

Figure 1

In the halfspace model of $\mathbb{H}^2$ with $E = \{x > 0,\ y > 0\}$:

$$h(x, y) = \ln\left(\frac{\sqrt{x^2 + y^2} + y}{x}\right) \quad x > 0,\ y > 0$$

Let K be a compact subset of D. Let α be a complete geodesic, disjoint from the geodesic containing A, intersecting C in two points, such that the region of D bounded by α and A is disjoint from K. The geodesic α separates the circle at infinity in two arcs; let B denote the arc at infinity such that the disk E, bounded by $\alpha \cup B$, contains K. Let h be the minimal graph on E which is $+\infty$ on α and on B it is the maximum of f on $K \cap C$. By the maximum principle, each u_n is bounded by h on K. Then, the sequence $\{u_n\}$ converges to a minimal solution u on D.

The existence of Scherk's type surface in a triangle, guarantees that u takes the right boundary values, as in [1]. □

4. Now we improve Theorem 4 in [4].

   (a) One can relax the hypothesis on the regularity of the curve Γ. It is enough to assume that it is rectifiable instead of C^0. Let us prove it. Assume that one has proved Theorem 4 for differentiable boundary values. Then, consider two families of differentiable curves approximating Γ, constructed as follows. For any $\varepsilon > 0$, let $\Gamma_\varepsilon^+ \subset \partial_\infty\mathbb{H}^2 \times \mathbb{R}$

be a differentiable vertical graph above Γ, such that the vertical distance between Γ and Γ_ε^+ is at most ε. Then, let $\Gamma_\varepsilon^- \subset \partial_\infty \mathbb{H}^2 \times \mathbb{R}$ be a differentiable vertical graph below Γ, such that the vertical distance between Γ and Γ_ε^- is at most ε.

For any $\varepsilon > 0$ there exist a minimal graph M_ε^+ (M_ε^-) with asymptotic boundary Γ_ε^+ (resp. Γ_ε^-).

By the maximum principle, each surface M_n in the proof of Theorem 4 in [4] is above M_ε^- and below M_ε^+, hence also the limit surface M. Now, let ε go to zero: as both Γ_ε^+ and Γ_ε^- converge to the curve Γ, the boundary of M must be Γ.

(b) The method of the proof in [4] is correct. There is a mistake in our boundary barrier, given by the graph of the function g. Instead of the graph of the function g, we use the Abresch-Sa Earp graph.

5. In Section 7 of [4], we neglegted some references. The original idea of the proof of Theorem 5 is from [7]. A removable singularities Theorem analogous to Theorem 5, for prescribed mean curvature graphs in Euclidean space and hyperbolic space is proved in [5] and [2] respectively.

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