

STRUCTAPP2020

*A two-day workshop on structure-preserving approximation
of evolutive problems and applications*

University of L'Aquila

23–24 January 2020

BOOK OF ABSTRACTS

Program at a glance

January 23		January 24	
10.00	Paternoster	9.30	Simoncini
10.30	Maset	10.15	Zennaro
11.00	<i>Coffee break</i>	11.00	<i>Coffee break</i>
11.30	Vermiglio	11.30	Poloni
12.00	Breda	12.15	Tudisco
12.30	Elia		
13.00	<i>Lunch</i>	13.00	<i>Lunch</i>
14.30	Boffi	14.30	Pugliese
15.15	Rozza	15.00	Viviani
16.00	<i>Coffee break</i>	15.45	<i>Coffee break</i>
16.30	Prato	16.15	Abdi
17.15	Rebegoldi	16.45	Rossi

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Thursday, 23 June 2020

Multivalued collocation methods free from order reduction

23 Jan
10:00

Beatrice Paternoster
University of Salerno

We introduce a theory of multivalued collocation methods in comparison with classical collocation based Runge-Kutta methods. The main issue we achieve by multivalued collocation methods is the lack of order reduction, which is typical of Runge-Kutta methods when applied to strongly stiff problems. This is due to the fact that multivalued collocation methods converge with uniform effective order of convergence on the overall integration interval, also in presence of stiffness. For stiff problems, Gaussian Runge-Kutta methods converge with order m , being m the number of internal stages, even if they have theoretical order equal to $2m$. It is also worthwhile mentioning that both multivalued and Runge-Kutta collocation methods require the same computational effort; moreover, stability properties are also the same, since both multivalued and Runge-Kutta collocation methods are A-stable. The theoretical analysis, examples of methods as well as numerical experiments on a selection of stiff problems are given in this talk. This is a joint work with Raffaele D'Ambrosio (University of L'Aquila).

Relative error analysis in numerical integration of linear ODEs: some surprising facts

23 Jan
10:30

Stefano Maset
University of Trieste

The error analysis in the numerical integration of ODEs is a well-known subject, but a better understanding of how this error grows with time can be obtained by comparing it with the growth of the solution, namely studying the relative error. Here, we consider as test equations linear ODEs with normal matrices. We show that, unlike the absolute error, the relative error always grows linearly in time and, in the long-time, contributions to the relative error coming from non-rightmost eigenvalues vanish.

Numerical approximation of the basic reproduction number in population models

23 Jan
11:30

Rossana Vermiglio
University of Udine

The basic reproduction number, simply denoted by R_0 , plays a fundamental role in the analysis of population and epidemic models. For many structured mathematical models, the linearization around an equilibrium can be written as a linear abstract differential equation on a Banach space, and R_0 is defined as the spectral radius of the infinite-dimensional next generation operator. Therefore suitable numerical methods are needed to compute it. Here we present the pseudospectral discretization (PSD) technique, which constructs an approximation of R_0 by turning the infinite-dimensional next generation operator into a matrix, and then by computing the spectral radius by eigenvalue algorithms for matrices. The efficiency and versatility of the PSD technique are confirmed by the experimental analysis of some examples. This research is in collaboration with D. Breda and F. Florian (University of Udine, Italy), J. Ripoll (University of Girona, Spain).

Periodicity, delays and numerical methods in biomathematics: a recent account

23 Jan
12:00

Dimitri Breda
University of Udine

Periodicity is common in natural phenomena. And delays are not so uncommon if we accept the challenge of dealing with complex, yet more realistic models. Numerical analysis is then mandatory when the target is either simulating, analyzing stability or detecting bifurcations. Based on the recent works [1,2,3], in this talk an up-to-date account is given of the interplay of these three characters, with an eye on applications in biomathematics. This is a joint work with Alessia Andò and Davide Liessi (University of Udine).

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Discretization of piecewise smooth differential systems

24 Jan
12:30

Cinzia Elia

University of Bari

We consider a model planar system with discontinuous right-hand side possessing an attractive periodic orbit and we investigate its persistence under numerical discretization. Well known results from the 80's on discretizations of smooth systems with hyperbolic periodic orbits insure the existence of an invariant curve for their numerical discretizations. For piecewise smooth systems there is no such curve in general, instead we show that there is an invariant band with band-width proportional to the discretization stepsize. We further consider an event-driven discretization of the model problem, whereby the solution is forced to step exactly on the discontinuity line. For this discretized system there is a periodic solution near the periodic orbit of the original problem (for sufficiently small discretization stepsize). Finally, we consider what happens to the Euler discretization of the scalar regularized system rewritten in polar coordinates, and give numerical evidence that the discrete solution now undergoes a period doubling cascade with respect to the regularization parameter.

A fictitious domain approach to fluid-structure interaction problems

23 Jan
14:30

Daniele Boffi

King Abdullah University of Science and Technology Kingdom of Saudi Arabia

We review a distributed Lagrange multiplier formulation of the Finite Element Immersed Boundary Method for the numerical approximation of the interaction between fluids and solids. The discretization of the problem leads to a mixed problem for which a rigorous stability analysis is provided. Optimal convergence estimates are proved for its finite element space discretization. The model, originally introduced for the coupling of incompressible fluids and solids, can be extended to include the simulation of compressible structures. Recent research investigates several time marching strategies for the proposed method.

Structure preserving reduced order methods for fluid-structure interaction parametric problems: state of the art and perspectives

23 Jan
15:15

Gianluigi Rozza

SISSA, Scuola internazionale Superiore di Studi Avanzati, Mathematics, mathLab

We describe the state of the art and perspectives in the developments of efficient structure preserving reduced order methods for parametric nonlinear fluid-structure interaction problems by monolithic and segregated approaches, as well as the use of numerical techniques to enhance the reduction of the Kolmogorov n-width in order to improve computational performances. Special attention is dedicated to the approximation stability of the reduced order model by supremisers, to the structure preserving property, as well as to the imposition of accurate and efficient coupling conditions to guarantee continuity of quantities at the fluid-structure interface. This is a joint work with Monica Nonino, Francesco Ballarin (SISSA) and Yvon Maday (Sorbonne Paris LJLL).

Interior point methods meet neural networks: an application to image deblurring

23 Jan
16:30

Marco Prato

University of Modena and Reggio Emilia

We propose a novel neural network to approach the image restoration problem. This architecture is inspired from an interior point proximal optimization algorithm, capable of imposing useful constraints on the sought solution. In particular, the network is composed of proximal steps alternated with convolutional structures that are able to estimate in an automatic manner the involved parameters, such as the regularization parameter, the steplength and the barrier parameter. This is one of the advantages offered by the proposed network with respect to variational methods traditionally employed in image restoration, for which the choice of parameters is performed either empirically or with suboptimal techniques. Also numerical experiments for image deblurring/denoising show that our network trained in a supervised fashion is much faster and leads to a better restoration quality than standard optimization methods.

Inexact variable metric linesearch based methods with applications to image processing

23 Jan
17:15

Simone Rebegoldi

University of Modena and Reggio Emilia

In this talk, we introduce a variable metric linesearch based method suited for minimizing the sum of a smooth (possibly nonconvex) term, plus a convex (possibly nonsmooth) function. The proposed approach alternates a gradient step on the smooth part followed by a proximal step on the nonsmooth part, and then ensures the sufficient decrease of the objective function by performing a linesearch procedure along the descent direction. Unlike other existing methods in the literature, the proximal operator may be computed with respect to a variable (possibly non Euclidean) metric, provided that the parameters defining the metric belong to compact sets. Furthermore, we allow for the approximate computation of the proximal operator by means of an implementable inexactness criterion. Notably, the proposed method can be easily extended to address block coordinate optimization problems, in which the convex part is given by a finite sum of convex terms defined on disjoint blocks of variables. We show that each limit point of the iterates sequence is stationary and we prove strong convergence to the limit point by assuming that the objective function satisfies the Kurdyka–Lojasiewicz inequality and the gradient of the smooth part is Lipschitz continuous. The proposed method is then applied to a wide collection of image processing problems, such as image deblurring/denoising, image compression and blind deconvolution, showing very promising results in terms of robustness, accuracy and efficiency when compared to other state-of-the-art methods.

Friday, 24 January 2020

Matrix equations. Application to PDEs

Valeria Simoncini
University of Bologna

24 Jan
09:30

Matrix equations have arisen as the natural setting for various PDE discretization methods such as finite differences, isogeometric analysis, spectral and finite elements. Thanks to major recent computational advances, solving certain classes of linear matrix equations is a competitive alternative to dealing with the large (vector) linear systems classically stemming from the aforementioned discretizations. In this talk we support these considerations with examples from the numerical treatment of possibly time-dependent PDE problems.

On the most stable switching laws of linear switched systems

Marino Zennaro
University of Trieste

24 Jan
10:15

We deal with discrete-time *linear switched systems* of the form

$$x(n+1) = A_{\sigma(n)} x(n), \quad \sigma : \mathbf{N} \longrightarrow \{1, 2, \dots, m\},$$

where $x(0) \in \mathbf{R}^k$, the matrix $A_{\sigma(n)} \in \mathbf{R}^{k \times k}$ belongs to a finite family

$$\mathcal{F} = \{A_i\}_{1 \leq i \leq m}$$

associated to the system and σ denotes the switching law.

It is known that the most stable switching laws are associated to the so-called spectrum-minimizing products of the family \mathcal{F} . Moreover, for a normalized family \mathcal{F} of matrices (i.e., its lower spectral radius $\check{\rho}(\mathcal{F}) = 1$), that share an invariant cone K , all the most stable trajectories starting from the interior of K lie on the boundary of the antiball of a so-called invariant Barabanov antinorm, for which a canonical constructive procedure is available (Guglielmi & Z., 2015).

For families \mathcal{F} sharing an invariant cone K , in this talk we show how to provide lower bounds to $\check{\rho}(\mathcal{F})$ by a suitable adaptation of the Gelfand limit in the framework of antinorms (Guglielmi & Z., 2019). Then we consider families of matrices \mathcal{F} that share an invariant multicone K_{mul} (Brundu & Z., 2018, 2019) and show how to

generalize some of the known results on antinorms to this more general setting (Guglielmi & Z., in progress). These generalizations are of interest because common invariant multicones may well exist when common invariant cones do not.

Computing the nearest stable matrix via optimization on matrix manifolds

24 Jan
11:30

Federico Poloni
University of Pisa

We focus on the problem of computing the nearest stable matrix, i.e., $\min_X \|A - X\|_F$, where X ranges over the matrices with eigenvalues in the left half-plane. This problem has been studied in the past by various authors and has applications in systems theory: numerical and modelling errors may produce an unstable system of differential equations that needs to be corrected to a stable one. Some of the difficulties of this problem come from working with the feasible set, which is highly non-convex, and from the fact that the minimizers X often have multiple zero (or purely imaginary) eigenvalues, which makes their numerical computation problematic. We describe a reformulation that transforms the problem into the form $\min_U f(A, U)$, where U ranges over the orthogonal (or complex unitary) matrices. This formulation seems very effective numerically, as techniques for optimization on matrix manifolds have been studied extensively in the past years and are quite robust in practice — see, for instance, the book by Absil, Mahony, and Sepulchre. In addition, the technique can be adapted to various other related problems. This is a joint work with Vanni Noferini (Aalto University).

Can we compute the norm of a matrix?

24 Jan
12:15

Francesco Tudisco
Gran Sasso Science Institute

Computing a matrix norm induced by vector norms N_1 and N_2 is a classical problem in numerical analysis. However, except for a few choices of N_1 and N_2 , computing such a matrix norm to an arbitrary precision is generally unfeasible for large matrices as this is known to be an NP-hard problem. The situation is different when the matrix has nonnegative entries: In this talk I will present conditions on the entries of a nonnegative matrix and on the norms N_1 and N_2 that ensure global convergence of a "nonlinear power method" to the corresponding induced matrix norm. The new conditions are sensibly weaker than what was previously known and allow us to consider quite sophisticated vector norms N_1 and N_2 . As a nontrivial application of this result, I will discuss a new lower bound for the logarithmic Sobolev constant of a Markov chain.

Coalescence of eigenvalues of Hermitian matrices: a perturbative approach

24 Jan
14:30

Alessandro Pugliese
University of Bari

In this talk we consider a perturbative approach to the problem of detecting coalescence of eigenvalues of Hermitian matrices depending on 3 real parameters. The approach goes through the following steps: (1) associating to the original Hermitian eigenproblem an equivalent real symmetric one, twice the size, with eigenvalues that are (at least) double; (2) perturbing the new problem so that its eigenvalues are expected to coalesce along curves; (3) inferring from the structure of those curves coalescence of the eigenvalues for the original problem. We will see how the steps above can be rigorously justified and numerically implemented to locate points in parameters' space where eigenvalues coalesce. The talk is based on joint work with Luca Dieci (Georgia Institute of Technology).

Lie-Poisson methods for isospectral flows and their application to long-time simulation of spherical ideal hydrodynamics

24 Jan
15:00

Milo Viviani
University of Gothenburg

The theory of isospectral flows comprises a large class of continuous dynamical systems, particularly integrable systems and Lie–Poisson systems. Their discretization is a classical problem in numerical analysis. Preserving the spectra in the discrete flow requires the conservation of high order polynomials, which is hard to come by. Existing methods achieving this are complicated and usually fail to preserve the underlying Lie–Poisson structure. Here we present a class of numerical methods of arbitrary order for Hamiltonian and non-Hamiltonian isospectral flows, which preserve both the spectra and the Lie–Poisson structure. The methods are surprisingly simple, and avoid the use of constraints or exponential maps. Furthermore, due to preservation of the Lie–Poisson structure, they exhibit near conservation of the Hamiltonian function. As an illustration, we apply the methods to long-time simulation of the Euler equations on a sphere. Our findings suggest that our structure-preserving algorithms, on the one hand, perform at least as well as other popular methods (i.e. CLAM) without adding spurious hyperviscosity terms, on the other hand, show that the conservation of the Casimir functions can be actually used to predict and characterize the final state of the fluid.

Second derivative general linear methods for ordinary differential equations

24 Jan
16:15

Ali Abdi
University of Tabriz

There has been a great deal of interest in deriving efficient numerical methods for the numerical solution of initial value problems for systems of ordinary differential equations (ODEs)

$$\begin{cases} y'(t) = f(y(t)), & t \in I := [t_0, T], \\ y(t_0) = y_0, \end{cases} \quad (1)$$

with $y_0 \in \mathbb{R}^m$ and $f : \mathbb{R}^m \rightarrow \mathbb{R}^m$. General linear methods (GLMs) as a unifying framework for the traditional methods, namely linear multistep (multivalued) and Runge–Kutta (multistage) methods, have been introduced by Butcher. Because of the advantages of using the second derivative of the solution in the algorithm and existing of some efficient methods with the second derivative within the two main classes mentioned above, GLMs were extended to second derivative general linear methods (SGLMs). In an SGLM for the numerical solution of the IVP (1) which can be represented using its coefficients matrices

$$\begin{aligned} A &= [a_{ij}] \in \mathbb{R}^{s \times s}, \quad \bar{A} = [\bar{a}_{ij}] \in \mathbb{R}^{s \times s}, \quad U = [u_{ij}] \in \mathbb{R}^{s \times r}, \\ B &= [b_{ij}] \in \mathbb{R}^{r \times s}, \quad \bar{B} = [\bar{b}_{ij}] \in \mathbb{R}^{r \times s}, \quad V = [v_{ij}] \in \mathbb{R}^{r \times r}, \end{aligned}$$

the quantities imported into and evaluated in step number n are related by

$$\begin{cases} Y^{[n]} = h(A \otimes I)f(Y^{[n]}) + h^2(\bar{A} \otimes I)g(Y^{[n]}) + (U \otimes I)y^{[n-1]}, \\ y^{[n]} = h(B \otimes I)f(Y^{[n]}) + h^2(\bar{B} \otimes I)g(Y^{[n]}) + (V \otimes I)y^{[n-1]}, \end{cases}$$

where $n = 1, 2, \dots, N$, $Nh = \bar{x} - x_0$, $g(y) := f'(y)f(y)$, and I is the identity matrix of dimension m . Here $Y^{[n]} := [Y_i^{[n]}]_{i=1}^s \in \mathbb{R}^{ms}$ is the vector of stage values which are approximations of stage order q to the solution y of (1) at the points $x_{n-1} + c_i h$, $i = 1, 2, \dots, s$, and also $f(Y^{[n]}) := [f(Y_i^{[n]})]_{i=1}^s \in \mathbb{R}^{ms}$ and $g(Y^{[n]}) := [g(Y_i^{[n]})]_{i=1}^s \in \mathbb{R}^{ms}$. Also, the vector of external values $y^{[n-1]} := [y_i^{[n-1]}]_{i=1}^r \in \mathbb{R}^{mr}$ denotes the vector of approximations imported into step n and $y^{[n]} := [y_i^{[n]}]_{i=1}^r \in \mathbb{R}^{mr}$ is the vector of the quantities computed in this step and exported for use in the subsequent step.

In this talk, the implementation of second derivative general linear methods (SGLMs) in a variable stepsize environment using Nordsieck technique is discussed and various implementation issues are investigated. The talk is based on the results in the publication:

A. Abdi, Implementation of second derivative general linear methods, submitted.

Federico Rossi

University of Siena

Synchronization between individual elements in biological systems often proceeds through chemical communication via the exchange of specific messenger molecules. On a lab scale, these phenomena can be modeled by encapsulating an oscillating chemical reaction, which serves as a signal (information) sender/receiver element, inside micro-compartments such as droplet emulsions, liposomes and polymersomes [1]. The Belousov-Zhabotinsky (BZ) reaction is a well-known chemical oscillator largely used as a model for complex nonlinear phenomena, including chemical, physical and biological examples [2]. When the BZ-reaction is encapsulated inside micro-compartments, its chemical intermediates can serve as messengers by diffusing among different single units, to trigger specific reactions leading to a collective behavior between the elements [1,3]. The droplets spatial configuration and the geometry of the diffusion pathways play here an important role in governing the collective behavior of the system. In this context, microfluidics is a versatile tool to encapsulate the BZ-reaction in monodisperse micro-compartments and also for creating geometries and networks with well-defined boundaries [1,3–5]. The individual compartments can be engineered with selected properties using different surfactants, in the case of simple emulsions, or with specific membrane properties in the case of liposomes [6–8]. Here, we show that the coupling mediated by lipid membranes can be tuned according to the membrane composition and/or geometrical configuration of the network. In a 2-dimensional arrangement, the communication was mostly mediated by activators and the resulting global dynamics was dominated by in-phase oscillations [9,10]. In contrast, 1-dimensional arrangements had mostly an inhibitory character and the global dynamics resulted in anti-phase oscillations of the adjacent droplets and in-phase oscillations of alternating droplets. For 1-D configurations, the communication could be controlled by the insertion of sodium tetradecylsulphate and cholesterol as membrane dopants [7]. Numerical simulations suggested that the hydrophobic properties and the lipid packing at the interface were of paramount importance for the trans-membrane crossing of the pertinent chemical species. This is a joint work with Marcello Budroni (University of Sassari), Kristian Torbensen (Sorbonne Universités), Sandra Ristori (University of Florence) and Ali Abou-Hassan (Sorbonne Universités).

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Posters

Computing periodic solutions for complex models in population dynamics

Alessia Andò
University of Udine

Periodic solutions of renewal equations (REs) can be approximated, within a continuation framework, using collocation methods. In this context periodic solutions are conveniently expressed as solutions of two-point BVPs. Collocation for REs has mainly been developed for IVPs so far, thus we investigate a method based on piecewise polynomials, following those introduced in [1] for delay differential equations (DDEs). The final aim is to extend it to coupled RE/DDE systems modeling, e.g., structured populations. Moreover, in view of a theoretical convergence analysis, we mention a possible alternative inspired by [2], where a general approach to solve BVPs for neutral functional differential equations numerically is proposed. Periodic solutions of non-neutral DDEs and REs, as well as coupled systems, can be expressed as solutions of a BVP in the required form. However, the standard formulation of the problem as a two-point BVP does not satisfy the necessary conditions for the convergence of the method. We suggest, therefore, a different formulation which allows us to overcome the problems above. This is a joint work with Dimitri Breda (University of Udine).

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Diagonally implicit multivalued collocation methods

Maria Pia D'Arienzo

University of Salerno

We present multivalued almost collocation methods with diagonal coefficient matrix for the solution of ordinary differential equations:

$$\begin{cases} y'(t) = f(y(t)), & t \in [t_0, T], \\ y(t_0) = y_0, \end{cases}$$

where $f : \mathbb{R}^k \rightarrow \mathbb{R}^k$. On the uniform grid $t_n = t_0 + nh$, $n = 0, 1, \dots, N$, with $Nh = T - t_0$, the method takes the form:

$$Y_i^{[n]} = h \sum_{j=1}^m a_{ij} f(Y_j^{[n]}) + \sum_{j=1}^r u_{ij} y_j^{[n-1]}, \quad i = 1, 2, \dots, m,$$
$$y_i^{[n]} = h \sum_{j=1}^m b_{ij} f(Y_j^{[n]}) + \sum_{j=1}^r v_{ij} y_j^{[n-1]}, \quad i = 1, 2, \dots, r.$$

Multivalued methods are characterized by two integers, r and m , which represent the numbers of internal and external stages, respectively, an abscissa vector and four coefficient matrices $\mathbf{A} = [a_{ij}]$, $\mathbf{U} = [u_{ij}]$, $\mathbf{B} = [b_{ij}]$ and $\mathbf{V} = [v_{ij}]$, which permit to compute the internal stages and, starting from them, the external ones.

These methods can be extended using collocation in order to obtain a smooth solution. Collocation is a technique which approximates the solution with continuous approximants belonging to a finite dimensional space (usually polynomials).

Because of the implicitness of such methods, the computational cost of the integration process is strictly connected to the numerical solution of non linear systems of stage values, so we focus our attention in the development of methods with diagonal coefficient matrix \mathbf{A} . This structured matrix permits to parallelize the method easily and so, to reduce the computational effort.

We prove that those methods have at least order $p = r - 1$ and we provide examples of A-stable methods with two and three stages and order 3. A-stability is verified using Schur criterion. Moreover, numerical experiments are presented, which confirm the theoretical order of the methods, even in case of stiffness. This is a joint work with Dajana Conte (University of Salerno), Raffaele D'Ambrosio (University of L'Aquila) and Beatrice Paternoster (University of Salerno).

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Nonlinear stability analysis for stochastic θ -methods

Stefano Di Giovacchino
University of L’Aquila

We focus our attention on the numerical discretization of nonlinear stochastic differential equations by means of stochastic θ -methods. In particular, we investigate their nonlinear stability properties with respect to nonlinear test problems such that the mean-square deviation between two solutions exponentially decays, i.e., a mean-square contractive behaviour is visible along the stochastic dynamics. We aim to make the same property visible also along the numerical discretization via stochastic θ -methods: this issue is translated into sharp stepsize restrictions depending on parameters of the problem, here accurately estimated. A selection of numerical tests confirming the effectiveness of the analysis and its sharpness is also provided for both scalar and vector valued problems. This is a joint work with Raffaele D’Ambrosio (University of L’Aquila).

A PDE-based parameter estimation for a trigonometric finite difference numerical scheme

Ahmed El Fauti
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We consider the numerical solution of reaction-diffusion equations of λ - ω type, which are known to possess a one-parameter family of periodic plane wave solutions. Due to the periodic character of such solutions, a special purpose numerical integration is here proposed, based on trigonometric finite differences. The coefficients of the corresponding scheme depend on a parameter (related to the frequency of the oscillations and the chosen stepsize) that needs to be accurately estimated. In this work, a procedure for the estimation is proposed, based on the numerical solution of a single additional scalar PDE, strongly depending on the original problem. Numerical experiments confirming the effectiveness of the approach are given. This is a joint work with Raffaele D’Ambrosio (University of L’Aquila).

Exponential fitting: user-friendly reformulation

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Exponential fitting is a procedure to generate numerical methods for different operations, such as interpolation, quadrature, or solution of differential equation, on function with a pronounced oscillatory or hyperbolic behaviour. The coefficients of the corresponding methods are functions of a parameter z , related to the solution of the problem and the chosen stepsize. Often, these coefficients have the indeterminate form $0/0$ when z tends to 0 and serious numerical cancellation problems. To solve these problems, we propose user-friendly versions of the coefficients, via the so-called CS-factorizations. This technique is applied to selected exponentially fitted methods and avoids series expansion of the coefficients, making the methods accurate for both small and large values of z . Numerical experiments are presented, in order to confirm the theoretical results on selected test problems. This is a joint work with Dajana Conte (University of Salerno), Raffaele D'Ambrosio (University of L'Aquila), Liviu Gr. Ixaru (Horia Hulubei National institute of Physics and Nuclear Engineering, Bucharest) and Beatrice Paternoster (University of Salerno).

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Stability of periodic orbits of delay equations

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We prove the validity of a Floquet theory and the existence of Poincaré maps for periodic solutions of renewal equations, also known as Volterra functional equations. Our approach is based on sun-star perturbation theory of dual semigroups and relies on a spectral isolation property and on the regularity of the semiflow. This contributes a new chapter to the stability analysis, in analogy with ordinary and retarded functional differential equations as well as the case of equilibria. This is a joint work with Dimitri Breda (University of Udine).

Adapter peer methods for oscillatory problems

Leila Moradi

University of Salerno

We present exponentially fitted two step peer methods for the numerical solution of systems of ordinary differential equations having oscillatory solutions [2,3,4]. Such equations arise for example in the semi-discretization in space of advection-diffusion problems whose solution exhibits an oscillatory behaviour, such as the Boussinesq equation [1]. Exponentially fitted methods are able to exploit a-priori known information about the qualitative behaviour of the solution in order to efficiently furnish an accurate solution. Moreover peer methods are very suitable for a parallel implementation, which may be necessary when the number of spatial points increases. The effectiveness of this problem-oriented approach is shown through numerical tests on well-known problems. This is a joint work with Dajana Conte (University of Salerno), Fakhroddin Mohamadi (University of Hormozgan, Iran) and Beatrice Paternoster (University of Salerno).

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Capture the past to portray the future. New prospects for the numerical bifurcation analyses of structured population models

Francesca Scarabel

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Using pseudospectral discretization, a nonlinear delay equation (including differential and integral equations) can be approximated with a system of ordinary differential equations, whose dynamics and bifurcations can be studied with available software packages. The technique is illustrated by means of several models from population dynamics. This is a joint work with Dimitri Breda (University of Udine), Odo Diekmann (University of Utrecht), Mats Gyllenberg (University of Helsinki) and Rossana Vermiglio (University of Udine).

An efficient method for non-negative low rank-completion

Carmela Scalone

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We propose a new method for low-rank completion of a large sparse matrix, subject to non-negativity constraint. As a prototype of this problem we mention the well-known Netflix problem. Our method is based on the derivation of a constrained gradient system and its numerical integration. Indeed the main method presents two levels, in the inner level we minimize a functional associated to the low-rank completion of smaller distance to the given matrix. The low-rank matrix is expressed in the form of the non-negative factorization $X = \varepsilon UV^T$, where the factors U and V are assumed to be normalized with $\|U\|_F = 1$ and $\|V\|_F = 1$. In the outer level we tune the parameter ε until we minimize the associated functional. Numerical experiments on well-known large test matrices show the effectiveness of the method when compared to other algorithms available in the literature. This is a joint work with Nicola Guglielmi (Gran Sasso Science Institute).

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