State Vector Collapse as a Classical Statistical Effect of Measurement

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1. INTRODUCTION

The absence of a general consensus among physicists about the old question of the measurement induced state vector collapse in quantum mechanics is, at first sight, puzzling. It is true, of course that the question cannot, at least as yet, be settled by submitting it to the final decision of experiment. Still, one would like to understand better the main issue of disagreement which leads to such a wide variety of possible solutions. We believe that the origin of this disagreement lies in a fundamental ambiguity which exists in our understanding of the relation between quantum mechanics (QM) and classical statistical mechanics (CSM). For this reason we will start by discussing at length this ambiguity, whose clarification is, in our opinion, preliminary to any attempt to find a solution of the question which is the central theme of our Colloquium. We will come back to this question, therefore, only in the final part of this talk.

We will start by asking a very simple question. A basic axiom of QM is that, if the state of a particle is the superposition of two states belonging to different values of a given dynamical variable, it is only in the act of measurement that the variable acquires, at random, one of these two values. On the other hand classical mechanics assumes that a variable has always a precise value whether we measure it or not. One may ask therefore how is it that, in the limit when QM tends to CSM, the statement that a given variable of a physical system has always a precise value independently of having been measured or not - a meaningless statement in QM - gradually becomes meaningful. In other words, how can it be that QM, which is a theory describing the intrinsically probabilistic properties of quantum objects, becomes, in this limit, a statistical theory describing a probabilistic knowledge of intrinsically well determined properties of classical objects?

The first thing to do, of course, is to define properly what we mean by the "limit when QM tends to CM". This procedure is not as simple as letting h go to zero. In fact h is a constant of nature and one should rather take the correct limit for the appropriate dynamical variables when they become large compared with the relevant atomic units. To this purpose, however, one should realize that there are at least two different cases in which one expects that QM should approach CM.

The first case refers to the limit of large "quantum numbers" for a given quantum system, in the sense of Bohr's correspondence principle. This is the limit in which the variation ΔS of the phase of the wave function within the region where it is substantially different from zero is much larger than h (in other words the limit in which the wave length of a wave packet is much smaller then its extension), or, if the state is stationary, the energy E_n is much larger than the ground state energy E_0. The question is now: when the particle is in a state of this kind can one still maintain that its variables are not defined before they have been measured? Or, more precisely, if the state vector of the particle is a superposition of two states corresponding to macroscopically different values of a given variable, can one still maintain that this variable acquires, at random, one of these values only when it is measured? This would be in contrast with the classical statistical picture which supposes that the macroscopic variable does have one of the two possible values independently of whether it has been measured or not.

The second one, the case of a macroscopic body, is even more puzzling. The system is now made of an enormous number N of elementary quantum systems and has a correspondingly large number of degrees of freedom. For such a system it is generally possible to define at least a pair of collective (pseudo)conjugate variables (e.g. the center of mass coordinate and its velocity) who satisfy two conditions: (a) the commutator of these collective variables vanishes as N goes to infinity; (b) they are decoupled from the variables of the individual particles, and their Heisenberg equations of motion tend, in this limit, to the classical equations of motion.

One might think therefore that in this case property (b) gives an answer to our question. Since the classical equations of motion can be solved with precisely given initial values for both these variables their value is completely determined at all later times. This means that these variables "have" precisely determined values whether they are measured or not.

This statement is, however, only a partial answer, because it is true only for the particular choice of initial conditions just considered. In fact, QM yields, as we shall see in more detail in a moment, a statistical distribution in the phase space of the collective variables that does not generally reduce to a single point. It may happen, for example, that a pure quantum mechanical state corresponds to a limiting classical statistical distribution having the form of two delta functions centered on two different classical trajectories. Do we have, here again, to assume that our macroscopic body choses, at random, one of the two possible trajectories (on each of which the center of mass position and velocity are defined at any time), only when a measurement is performed on it? Should we not rather assume, in view of the fact that the body is macroscopic, that the right description is given by CSM, and that the double delta function simply reflects our ignorance?

In the following we will discuss this puzzling question showing that if we choose the second answer, consistently with the physical content of CM, we are led also to admit that the widely spread belief in the current interpretation of QM needs revisiting. If we do not wish to do so we have to adopt a strictly subjective conception of the classical world outside us, implying that also macroscopic objects are not localized anywhere before we look at them.

2. THE CLASSICAL LIMIT OF A QUANTUM STATE

We shall now discuss the statistical properties of a quantum state in the classical limit. As is well known, Wigner (1932) has defined a quantum mechanical function W(x,p) by means of the relation (for simplicity we put $h = 1$, but we will later introduce, when necessary, explicitly h):

$$W(x,p) = \pi^{-1} \int \psi^*(x+y)\, \psi(x-y)\, \exp(2ipy)\, dy \qquad (1)$$

This function may be used to compute the expectation value of any quantum variable $\mathbf{A}(\mathbf{x},\mathbf{p})$ function of the operators $\mathbf{x}$ and $\mathbf{p}$, by means of an expression:

$$\langle \mathbf{A} \rangle \equiv \int \psi^*(x)\, \mathbf{A}(x, -i\, \partial/\partial x)\, \psi(x)\, dx = \iint W(x,p)\, A(x,p)\, dx\, dp \qquad (2)$$

provided one takes

$$A(x,p) = 2 \int \langle x+z|\mathbf{A}|x-z\rangle \exp(-2ipz)\, dz. \tag{3}$$

W(x,p) corresponds to the classical distribution function f(x,p) in phase space because eq. (2) is formally identical to the classical expression

$$\langle A\rangle = \iint dx\, dp\, A(x,p)\, f(x,p). \tag{4}$$

where A(x,p) is the classical variable which corresponds to **A(x,p)**.

Of course, W(x,p) is not everywhere positive as the classical f(x,p) and therefore can not be interpreted as a distribution function. Nevertheless eq.(2) reduces, generally, to eq.(4) in the classical limit. In the second part of this section we will show with a few examples that this limit consists essentially in considering large quantum numbers.

On a formal level one can simply suppose that h is negligibly small($h \to 0$). In this case $A(x,p) \to A(x,p)$, when the quantum variable **A(x,p)** has the same form of the classical variable A(x,p) because its matrix elements $\langle x|\mathbf{A}|p\rangle$ differ from it only by terms of order h arising from the reordering of the operators **x** and **p** . On the other hand it is well known that the time evolution of W(x,p;t) reduces to the Liouville equation

$$\partial W(x,p;t)/\partial t = -(p/m)\, \partial W(x,p;t)/\partial x + (\partial V(x)/\partial x)(\partial W(x,p;t)/\partial p) \tag{5}$$

when the potential V(x) is slowly variable.

This reduction is a necessary step in order to obtain the classical distribution in the limit $h \to 0$. One needs in fact:

$$\int_A dx\, dp\, W(x,p) \;\to\; \int_A dx\, dp\, f(x,p)$$

for a finite region A of the phase space. This limit obviously holds when W(x,p) tends to the corresponding f(x,p).

An interesting particular case is that of a free particle wave packet initially concentrated in a Gaussian region:

$$\psi(x,0) = (\pi\alpha)^{-1/4} \exp\{-(x-x_o)^2/2\alpha\} \tag{6}$$

The energy expectation value ε is given by

$$\varepsilon = \hbar^2/4\alpha m \tag{7}$$

and the proper classical limit is provided by the limit $\varepsilon\, t >> \hbar$. Now one has:

$$W(x,p;t) \rightarrow (4\varepsilon t^2/m)^{-1/2} \exp[-m(x-x_o)^2/4\varepsilon t^2/ \delta[p-(x-x_o)m/t] \tag{8}$$

This is the classical distribution function f(x,p;t) in phase space of a free Hamiltonian fluid concentrated initially (t=0) at $x=x_0$.

When the state is stationary with energy $E >> E_o$ (ground state energy) one has

$$W(x,p) \rightarrow N\, p_E^{-1}[\, \delta(p-p_E) + \delta(p+p_E)] = N\, \delta(H-E) \tag{9}$$

where

$$H = (p^2/2m) + V(x) \tag{10}$$

and N is the phase space volume on the energy shell. Eq.(9) shows that the statistical properties of the quantum mechanical density matrix for a given energy E tend to those of the corresponding microcanonical ensemble of classical statistical mechanics.

Let us now see what happens when the state is a superposition

$$\psi = c_1\psi_1 + c_2\psi_2. \tag{11}$$

The total Wigner function will be the sum of the two Wigner functions of ψ_1 and ψ_2, weighted with the respective probabilities, plus an interference term, whose general features will be better understood by considering two complementary cases.

The first one is when the two wave functions are localized in two separate space regions. Then the contribution to W(x,p;t) of the interference term contains a factor

$$\cos\, [p(x_1-x_2)/\hbar\,] \tag{12}$$

where x_1 and x_2 are the mean values of x in ψ_1 and ψ_2. The contribution of this term to the expectation value of any variable A(x,p) vanishes unless its p-dependence shows the same rapidly oscillating behaviour of the cosine factor. This is certainly not the case for the quantum variables considered here that have a classical limit.

The complementary case obtains when the two wave functions are not localized in separate space regions, but are labeled with two widely different values of the energy $E_1 >> E_2 >> E_0$ (ground state energy). Here again the interference term gives a vanishing contribution to expectation values of variables which have a classical limit. This can be easily seen in the simple example of a particle constrained between two fixed boundaries in which the interference term to the Wigner function oscillates with a factor

$$\cos\{[\sqrt{(2m E_1)} - \sqrt{(2m E_2)}]x/\hbar\}. \qquad (13)$$

Here the contribution of this term to the expectation value of a variable A(x,p) is negligibly small unless its x-dependence shows the same rapidly oscillating behaviour of the cosine factor, a property which we do not expect a variable with a classical limit to possess.

It is therefore clear that the classical statistical ensemble corresponding to the quantum state (11) is always simply the union of the two classical statistical ensembles corresponding to the individual states, weighted with probabilities $|c_1|^2$ and $|c_2|^2$.

The case of a macroscopic body made of N particles coupled to each other may be treated by means of simple models and leads to exactly the same conclusions. The wave function of the system may be written in the form

$$\Psi = \phi(q)\ \chi\ (q_1, \dots q_{N-1}) \qquad (14)$$

where ϕ is the center of mass wave function and χ is the wave function of the N-1 independent relative coordinates.

Consider for the moment the q-dependent part of ψ. If we take for ϕ the form (6), the classical limit of its Wigner function, of the form (8), is now attained as $N \to \infty$. We might instead consider however, any other wave function of the center of mass. A stationary state of energy $E >> E_0$ would, for example, lead to a Wigner function which tends to (9) as $N \to \infty$. In other words the statistical properties of $\phi(q)$ are the same as those discussed in the previous section in the corresponding classical limit. The presence of the remaining variables does however make a difference in the case of a macroscopic body.

To see how this comes out let us consider a state described by a superposition of two wave functions of the form (14)

$$\Psi = c_1\phi_1\chi_1 + c_2\phi_2\chi_2 \tag{15}$$

The proper Wigner function of the intensive center of mass variables q,v is now constructed by integrating over all the microscopic variables. This leads to cross terms which are not only small for the reasons that made small the interference contribution (12) to the Wigner function derived from the wave function (11), but also because it is extremely unlikely that χ_1 and χ_2 are exactly the same. In general χ_1 and χ_2 are very different microscopically, namely orthogonal, even if they are macroscopically equivalent, and the cross terms actually vanish.

We may conclude therefore that there is always a one-to-one correspondence between a quantum mechanical state and a statistical distribution in phase space of classical statistical mechanics both in the case of a microscopic system and of a macroscopic body. The difference is that in the former the classical limit is attained when a suitable action variable is >> h, while in the latter the limit $N \to \infty$ is sufficient to ensure that the center of mass variables always behave as classical. We shall now discuss the consequences of this correspondence.

3. STATISTICAL PROPERTIES OF A QUANTUM STATE

It is now easy to discuss the statistical properties of a quantum mechanical state in the classical limit. It is particularly interesting to see what happens to the uncertainty product $\Delta x \ \Delta p$. For the free wave packet of eq.(6) one has:

$$(\Delta p)^2 = h^2/\alpha = 2\,\varepsilon m \tag{16}$$

$$(\Delta x)^2 = 2\,\varepsilon t^2/m + h^2/8\varepsilon m = (\Delta p)^2 t^2/m^2 + h^2/4(\Delta p)^2 \tag{17}$$

$$(\Delta x\,\Delta p)^2 = 4\,\varepsilon^2 t^2 + h^2/4 \to (2\,\varepsilon\, t)^2 \qquad \text{for } \varepsilon\, t >> h \tag{18}$$

Eq.(17) resembles closely an old relation which marked a turning point in the history of physics: Einstein's formula for the energy fluctuation of radiation at thermal equilibrium expressed as the sum of two terms of different origin (Einstein 1909). In the case of radiation the quantum term arises from its particlelike properties and the classical term from the wavelike ones. In quantum mechanics the reverse happens. In our case the first (particle) term has a classical origin and the second (wave) term a quantum one. This separation, however, has been forgotten since the adoption of the standard interpretation of QM, which considers the fluctuations of the quantum variables as wholly due to their intrinsically undetermined nature. What we propose, on the contrary, is to take seriously this separation as physically meaningful. From this point of view, eq.(17) means that the spread of a quantum wave packet for large values of εt does not arise from an ontologically intrinsic delocalization of the particle, but, as it happens for classical particles, is a trivial consequence of the fact that the region where a particle may be found increases with time if its momentum is not precisely determined.

Stated differently, eq. (18) indicates that the really intrinsic quantum indeterminacy, reflecting the impossibility of simultaneous existence of position and momentum, is always the minimum one implied by the Heisenberg principle. Higher indeterminacies are instead of statistical nature, reflecting the actual displacement in space of particles with different momenta,and they survive in the classical limit. This picture is particularly relevant for the interpretation of the superposition (11) when the two states are macroscopically different. One can no longer say that only when a variable is measured it assumes a value within one of the two ranges allowed by either ψ_1 or ψ_2.

Since the statistical ensemble described by the density matrix of the state (11) is, in the limit of large values of the action, the weighted union of two disconnected classical phase space distributions, we are almost forced to conclude that it is meaningful to say that the particle belonged to one or the other distribution even before the measure had taken place. We will explain and justify this statement in the following sections, showing that it is indeed possible to give a more precise meaning to the separation between quantum and classical indeterminacy.

4. ALTERNATIVE INTERPRETATIONS OF CLASSICAL STATISTICAL MECHANICS

Assume now that a classical distribution function in phase space is at t= 0 of the form:

$$f(q,p;0) = P_1 f_1(q,p;0) + P_2 f_2(q,p;0) \tag{19}$$

with $f_1=0$ when $q,p \notin \Gamma_1^o$ and $f_2=0$ when $q,p \notin \Gamma_2^o$ in phase space, with $\Gamma_1^o \cap \Gamma_2^o = 0$. Call q_i^o, p_i^o the mean values of q, p in the distribution f_i and Δq_o, Δp_o their mean square values, which we assume for simplicity to be the same for f_1 and f_2. Suppose furthermore that the space distance d_o between Γ_1^o and Γ_2^o is >> Δq_o. Now we measure q with a resolution Δq_o and find the particle in S_1^o, the space width of Γ_1^o. We might as well have measured p with a resolution Δp_o, with the result that we would have found the particle in M_1^o, the momentum width of Γ_1^o. In classical mechanics of course both measurements are compatible, but one is sufficient, in this case, to deduce from (19) that at t=0 the point in phase space representing the particle's state is in Γ_1^o. Then we have two possible interpretations of this fact:

(a) we can say that even before our measurement at an earlier time t the phase space point of the system was in Γ_1^t (the region which subsequently evolved according to Liouville into Γ_1^o at t=0) because it has followed a trajectory which, starting from a point located within Γ_1^t goes through a point in Γ_1^o. The position of the particle at the earlier time was therefore within a distance $\Delta q_t << d_t$ from the mean value q_1^t given by $f_1(q,p;t)$, with Δq_t given by (suppose for simplicity that the particle has propagated freely such that Δp_o does not vary with time):

$$\Delta q_t = [(\Delta q_o)^2 + (\Delta p_o t/m)^2]^{1/2} \tag{20}$$

It should be stressed at this point that, while the volume of Γ_1^t is equal to the volume of Γ_1^o, the uncertainty product $\Delta q_t \, \Delta p_t$ always increases with time (both in the backward and in the forward direction) because of (20).

The probabilities P_1 P_2 in (19) represent therefore our ignorance about the previous localization of the particle and not an actual indetermination of its position in space.

(b) we can say that before the measurement there was no phase space point representing the particle's state in $\Gamma_1{}^t$ or in $\Gamma_2{}^t$ and that therefore the state has been localized in $\Gamma_1{}^o$ by the measurement. In this case P_1 and P_2 are intrinsic probabilities of localizing the particle either in $\Gamma_1{}^o$ or in $\Gamma_2{}^o$. There is no trajectory followed by the particle from one point of $\Gamma_1{}^t$ to a given point of $\Gamma_1{}^o$.

In both cases, after the measurement the state is no longer represented by the distribution function f(q,p) but is reduced to $f_1(q,p)$, the new state created by the measurement, which evolves successively according to the Liouville equation. However, in the first case the state $f_1(q,p)$ is the state of a new ensemble in which the states of the individual particles are known only within the corresponding uncertainties, but in the second case there is no difference between the state of the particles and the state of the ensemble. Therefore one immediately recognizes that (a) is the usual interpretation of statistical mechanics in terms of classical dynamics, and (b) is an interpretation which closely resembles the conventional interpretation of quantum mechanics in which the observer has an essential role. In spite of the fact that they both lead to the same observable consequences, our choice is biased in favour of the first one by our belief in the existence of an objective world outside our mind.

Let us now consider the corresponding situation in quantum mechanics. Take a state defined by the wave function

$$\psi = \sqrt{P_1}\,\psi_1 + \sqrt{P_2}\exp(i\phi)\,\psi_2 \qquad (21)$$

whose Wigner function tends in the classical limit to (19)

$$W(q,p;0) \rightarrow f(q,p;0) \qquad (22)$$

The wave functions ψ_1 and ψ_2 have therefore the same mean values and mean square values of q and p as before. Let us assume Δq_o to be related to Δp_o by the minimum uncertainty:

$$\Delta q_o \Delta p_o \approx h/2. \qquad (23)$$

Suppose we measure q with the resolution Δq_o and find the particle within the space region $S_1{}^o$ which is the space support of ψ_1. We might as well have measured p with resolution Δp_o, with the result that we would have found the momentum of the particle in $M_1{}^o$, the momentum support of ψ_1. In both cases we deduce that the state of the particle at t=0 is represented by ψ_1,and evolves subsequently according to the Schrödinger equation. It should be stressed that also in the quantum case the two measurements are compatible, because the two resolutions satisfy the uncertainty relation (23). Both these measurements, therefore, reduce the state (21) but do not change the form of ψ_1. However, according to the conventional interpretation of quantum mechanics, we cannot infer, from this fact that the particle was in $S_1{}^t$ (or $M_1{}^t$) at an earlier time t, because we have to accept that the particle has been located in that region by the act of measurement, and that any statement about its position (or momentum) before the measurement is actually meaningless. Eq.(22), however, forces us to extend this interpretation also to classical statistical mechanics and therefore to adopt interpretation (b), because $S_1{}^t$ ($M_1{}^t$) is the space (momentum) extension of $\Gamma_1{}^t$. *We find therefore an inconsistency if we insist to accept the conventional interpretation (a) for classical statistical mechanics while retaining the conventional interpretation of quantum mechanics.*

5. CONSISTENCY REQUIREMENT BETWEEN CLASSICAL AND QUANTUM INTERPRETATIONS

The standard interpretation of quantum mechanics is therefore incompatible with the usual assumption that newtonian dynamics for individual particles underlies the description of classical statistical ensembles. This suggests that the introduction of the notion of a sort of localization of particles in space should implement the conventional formulation of quantum mechanics. This localization, of course, should always be consistent with the minimum uncertainty allowed by the Heisenberg principle. In other words we believe that one may describe the time evolution of a particle's state in terms of a sort of fuzzy trajectory which is undefined within the region of minimum uncertainty, but is sufficiently localized in phase space to exclude that it may instantaneously jump from one small region to another one very far away.

We are not going to construct explicitly a new theory of this sort. We wish however to examine in more detail whether the possibility exists of modifying the standard interpretation of quantum mechanics in order to save our traditional picture of classical mechanics.

We have dealt up to here with the problem of giving a meaning to the statement that a particle was localized into one or the other of two widely separated regions in space even before an actual measurement of its position has been made. In this case the resolution Δq_o is given by the width of each wave packet at the time of measurement. Suppose now one localizes a particle in a space region of extension Δq_o around a value q_o within a wave packet of larger extension. Does it still make sense to ask the question: where was the particle (again supposed to propagate freely) at an earlier time t?

The answer requires a brief discussion of the analogous classical case. Given a distribution function f(q,p,0) in phase space with mean square values $\Delta \underline{q}$, $\Delta \underline{p}$ of q and p, we can reduce our ignorance by measuring both q and p with resolutions Δq_o, Δp_o such that their product is much smaller than the product $\Delta \underline{q}\, \Delta \underline{p}$. Of course in classical mechanics we may chose these resolutions as small as we like (or at least as small as our instruments allow us to do so). Eq.(20) will therefore again give us the uncertainty of the position of the particle at an earlier time t, in terms of the values chosen for these resolutions.

We may now go back to the quantum case described by a wave function ψ whose Wigner function tends to f(q,p;0) in the classical limit. The uncertainties in q and p given by ψ are now such that $\Delta \underline{q}\, \Delta \underline{p} \gg h$. Again we may reduce our ignorance by measuring q and p with resolutions $\Delta q_o\, \Delta p_o$ such that their product is $\ll \Delta \underline{q}\, \Delta \underline{p}$ but, of course, we cannot make them as small as we like because of the minimum uncertainty relation (23). These ideal measurements however can be performed in such a way as to minimize the uncertainty in the position of the particle (again supposed to propagate freely) at an earlier time t. We obtain from (20), with the replacement $\Delta p_o = h / 2\, \Delta q_o$,

$$\Delta q_t = \min\, [(\Delta q_o)^2 + (h\, t/2m\Delta q_o)^2]^{1/2} \qquad h/2\, \Delta \underline{p} < \Delta q_o < \Delta \underline{q} \qquad (24)$$

where the minimum is taken with respect to Δq_o and depends on t.

For small times one has $\Delta q_t \approx h / 2m\, \Delta \underline{q}$; for intermediate times $\Delta q_t \approx (h\, t/2m)^{1/2}$, and for large times $\Delta q_t \approx h\, t/2m\, \Delta \underline{q}$.

This result shows that, even if we cannot precisely localize the particle on a trajectory as in classical mechanics, it is still possible to give an upper limit for the extension of the region where the particle was localized before the measurements. This statement, of course, does not conflict in any way with the physical predictions of quantum mechanics, but leads to the correct newtonian trajectories when the classical limit is performed.

In order to understand fully the meaning of our point of view, we stress again that the effect of a measurement which reduces a wave packet with uncertainty product $\Delta q \Delta p >> h$ into a wave packet with uncertainty $\Delta q_o \Delta p_o \approx h$, is substantially different from a change in the form of a wave packet which maintains the uncertainty equal to its minimum value. It is very important to avoid confusions between the two. The first one is irreversible, because our knowledge changes irreversibly. It implies, exactly as it does in classical mechanics, the measurement of both q and p, the only difference with classical mechanics being that now the resolutions Δq_o and Δp_o must satisfy the minimum uncertainty relation. The state of the individual particle is not reduced: it is only the ensemble state which is reduced. This measurement eliminates the "empty waves" of a superposition because they are not physical: they only represent our ignorance before the measurement.

The second change is reversible, because it corresponds to an actual change of the individual particle's physical state from a wave packet with $\Delta q'_o \Delta p'_o \approx h$ to a wave packet with $\Delta q_o \Delta p_o \approx h$ due to its Schrödinger evolution in presence of a physical interaction. Clearly, there is no reduction in this case, because there is no change in the information we have on the properties of the individual system: what we gain in the definition of q (if $\Delta q_o < \Delta q'_o$), we lose in the definition of p ($\Delta p_o > \Delta p'_o$) and viceversa.

6. THE STATE VECTOR COLLAPSE

We may now explicitly come to the problem of state vector collapse. In the usual formulation (D'Espagnat 1976), a microsystem S, whose state $|\psi>$ is a superposition of eigenstates $|\phi_n>$ of a quantum variable **G** with eigenvalues γ_n, is brought in interaction with a macroscopic measuring instrument M, made of a very large number N of particles whose states $|n>$ are labeled by the eigenvalues γ_n of a macroscopic physical variable Γ, establishing in this way a one-to-one correspondence between the set of γ_n and the set of γ_n. The state W of the total system $S+M$ is therefore after the interaction

$$\Omega = \Sigma_n c_n |\phi_n> |n> \qquad (25)$$

The corresponding density matrix can be written as

$$\rho_\Omega = \Sigma_{nn'} c_n c_{n'}^* \mid \phi_n\rangle\langle\phi_{n'}\mid \quad \mid n\rangle\langle n'\mid \tag{26}$$

Suppose now that the Wigner functions $W_n(q,v)$ of the intensive center of mass variables q, v of *M* constructed with the states $\mid n\rangle$ tend to the set of phase space classical distributions $f_n(q,v)$ which correspond to the disconnected phase space regions in which Γ has the different values γ_n. From the discussion at the end of section 2 (eqs. (16)-(18)) we can say that

$$W_n(q,v) \rightarrow f_n(q,v) \qquad N\rightarrow\infty \tag{27}$$

and, furthermore, that the contribution of the cross terms with $n\neq n'$ in (26) to W(q,v) have an oscillating behaviour of the form

$$K \cos[(\gamma_n - \gamma_{n'})\alpha] \tag{28}$$

where α is the variable conjugate to Γ and K is a slowly varying function of the variables q,v. These contributions therefore vanish in the limit $N\rightarrow\infty$ for the same reasons that made the cross term of the Wigner function of the state (15) vanish in the same limit. The statistical properties of the system *S+M* described by the density matrix (26) are therefore identical to those described by the reduced density matrix

$$\rho_s(q,v) = \Sigma_n \mid c_n\mid^2 f_n(q,v) \mid\phi_n\rangle\langle\phi_n\mid \tag{29}$$

which is a classical distribution function in the phase space of *M* and a density matrix for the variables of *S* . The meaning of (29) is of course that, from a statistical point of view, everything happens as if the wave packet reduction had occurred for the quantum variables of *S* after the interaction with *M* (Cini 1983).

The main objection to consider this line of reasoning as a satisfactory solution of the wave packet collapse problem is that eq.(29) represents only the properties of a statistical ensemble of systems *S+M* but does not represent the actual outcome of each individual act of measurement. In other words it does not explain why, after the interaction with *S* , the instrument *M* should objectively be in a given state with a well determined value, say γ_k, of

Γ independently of whether it is observed or not, while the superposition (25), according to the standard interpretation of quantum mechanics, collapses in a well determined state characterized by the value γ_k of Γ only when M, in its turn, is observed.

It is clear that the answer to this objection is provided by our interpretation of quantum mechanics. If our point of view is correct the minimum uncertainty product is completely negligible for the variables q and v of M and all the uncertainty arising from the sum over n in eq.(29) is of statistical nature, namely arises from our ignorance of the actual state of M, which, however, does have a definite value of Γ, independently of our knowledge. The reduction of the mixture (29) to the single term

$$f_k(q,v) \quad |\phi_k><\phi_k| \tag{30}$$

is only a matter of reduction of ignorance, not a physical phenomenon in which M is involved.

To illustrate the situation with a practical example let us consider a Stern Gerlach device in which a beam of particles initially located in a given space region is split, by means of an inhomogeneous magnetic field interacting with the magnetic moment of each particle, in two beams whose separation increases with time. We study the time evolution of their lateral widths which at t=0 coincide. The initial wave packet

$$\Psi(x,0) = \psi(x,0) \ [c_+u_+\exp(ip_ox/\hbar) + c_-u_-\exp(-ip_ox/\hbar)] \tag{31}$$

(where $\psi(x,0)$ is given by (10) and $u_\pm$ are the spin up and down eigenfunctions) gives rise, after a time t, to a mixed density matrix which is the sum of two widely separated parts:

$$\rho_s(x,p;t) = P_+W_+(x,p;t) \ |u_+><u_+| + P_-W_-(x,p;t) \ |u_-><u_-| \qquad P_\pm = |c_\pm|^2 \tag{32}$$

where $W_\pm(x,p;t)$ are given by eq.(8) with x_o replaced by $x_\pm = x_o \pm p_ot/m$. The interference terms are easily found, as already discussed at length, to be vanishingly small because the overlap of the two gaussians centered at $x_\pm$ is practically zero. The mean square values of x, p are given by (we take the trace on the spin variables):

$$(\Delta x)^2 = (\Delta x_o)^2 + (\Delta p_o)^2 t^2/m^2 + 4\, P_+ P_-\, p_o^2 t^2/m^2 \qquad (33)$$

$$(\Delta p)^2 = (\Delta p_o)^2 + 4\, P_+ P_-\, p_o^2 \qquad (34)$$

Now, if the lateral width of the individual beams is much smaller than their distance (namely if $p_o >> \Delta p_o$, and $t >> h\, m/p_o \Delta p_o$) the uncertainty product reduces to the classical expression

$$(\Delta x\, \Delta p)_{cl} = 4\, P_+ P_-\, p_o^2 t/m \qquad (35)$$

which represents the effect of the uncertainty $\pm p_o$ in the momentum of a particle on the spread of its position's uncertainty. Here again, if we find the particle in the region occupied by the beam with momentum $+p_o$, we have to conclude that it was in that beam even before we made the measurement. The probabilities $P_{\pm}$ represent therefore our ignorance and not an intrinsic delocalization of the particle.

The case of the Stern Gerlach device shows therefore that our interpretation and the conventional one have very different implications. For us the particle is already in one of the two beams before its detection by a counter which may have been placed on its way. The counter is discharged because the particle is already in the beam which impinges on it. We stress that this does not imply that coherence has been destroyed once for all. In fact, if the two beams are superimposed again, the occupied phase space is not anymore the union of two classically separated regions, and therefore the typical quantum interference occurs again. In our picture the reduction of the wave function is simply a consequence of the additional information acquired on the state of the particle which allows us to change our description of it, and no problem arises.

For the conventional interpretation it is the other way round: the counter's discharge localizes the particle in the region where it occurs, inducing an abrupt change in the physical entity represented by the wave function. In this case, as already discussed at length, one has to explain many puzzling features of this sudden and irreversible change of the particle's properties.

Obviously, once the reduction of the ensemble's state due to a reduction of ignorance is accepted as a feature of quantum mechanics which it has in common with classical statistical mechanics, the absence of reduction of the individual states will not lead to a nightmarish multiplication of worlds (Everett 1957), because the reduction of ignorance is sufficient to eliminate the proliferation (which, by definition, implies a tremendous increase with time of the uncertainty product) of branches of a composite system's wave function. At the same time this absence of reduction is sufficient to eliminate the extremely "counterintuitive mutual involvement of physical and mental phenomena" (Shimony 1963) invoked explicitly by von Neumann but implicitly accepted by all theories of measurement which adopt the wave function collapse postulate as a physical irreversible phenomenon which cannot be reduced to the Schrödinger time evolution.

The interpretation of quantum mechanics proposed here has, on the other side, some similarities with the so called "environmental" theories (Primas 1981) (Zurek 1983) (Joos and Zeh 1985) which attribute to a quantum object the capability of acquiring classical properties as a consequence of its interaction with the environment. They have in common with our approach in a broad sense the idea that quantum objects may acquire, in appropriate conditions, localization properties which are preexistent to their measurement.

There is however an essential difference. For these theories the interaction between instrument and environment eliminates the quantum correlations between the object and the instrument without introducing further correlations between the instrument and the environment, because the state of the latter is never detected. For us the quantum correlations between object and instrument are already washed out by the classical behaviour of the instrument for values of the macroscopic variables which do not show quantum interference effects. Since there are different versions of the "environmental" point of view a detailed comparison would be exceedingly lengthy. However we will attempt to extract their common feature.

In one version (Zurek 1983) it is the instrument M which interacts with the environment E. The initial state Ω of eq.(34) is therefore transformed into a state Ψ of the total system S+M+E. By taking the trace over the environment's variables which are not observed one obtains a reduced density matrix

$$\rho^{\circ}{}_{S+M} = \mathrm{Tr}_E\,(\Psi\Psi^{\dagger}) \qquad (36)$$

of the system S+M in which the off diagonal elements of the density matrix ρ_Ω constructed with the state Ω are absent. The off-diagonal elements of ρ_Ω are however already vanishingly small for all the macroscopic variables of M and there seems no need of further eliminating them.

In another version (Joos and Zeh 1985) it is the system *S* which is correlated with the environment. In this case the initial state *S+E*

$$\Phi = \Sigma_\alpha a_\alpha |\varphi_\alpha> |E_\alpha> \quad (37)$$

becomes, after the interaction between *S* and *M*

$$\Psi = \Sigma_{\alpha n} a_\alpha P_n |\varphi_\alpha> |E_{\alpha n}> |n> \quad (38)$$

where the environment states will be mutually orthogonal in each index. Again, taking the trace over *E* one obtains a reduced density matrix of the type (36).

In any case the main problem is still there, namely, to justify why we should interpret the density matrix (36) as a classical statistical ensemble, in which each individual system of the ensemble is in a definite, albeit unknown, state, rather than the density matrix of a quantum statistical mixture, in which each individual system is projected into a definite state only when it is observed. The difficulties that still remain in this approach as a consequence of this ambiguity are particularly evident in the Stern Gerlach example discussed above.

In this case one has actually, three variables of different nature. The first variable is the particle's spin, an essentially quantum variable. The second one is the particle's position, which is a quantum variable which may acquire macroscopic values. The third is the counter's charge which is a typical macroscopic variable. The spin, obviously, cannot be reduced to a classical variable by the interaction with the environment. Therefore, it is either the particle's position or the counter's charge which acquires classical properties.

In the first case, however, the environment which induces the collapse of the superposition into one or the other beam when the beams are far apart, is unable to transform back the mixture into a superposition when the beams are brought to overlap again. Since we know that if this happens they do interfere again, it seems difficult to ascribe to the environment the reversible space localization of the two beams that we have envisaged in order to reconcile QM with its classical limit.

In the second case the collapse of the superposition between the two macroscopically different states of the counter (neutral and discharged) may well be ascribed to the environment. In this case, however, one is again forced to assume, as in the standard interpretation, that the particle jumps from one beam to another, even when they are far apart, according to whether the counter is triggered or not by the interaction with the environment. This is indeed a counterintuitive conclusion that we prefer to avoid, if possible.

We think therefore that the elimination of the contradiction between the standard interpretation of quantum mechanics and the accepted interpretation of classical statistical mechanics yields also the solution of the old problem of the measurement induced state vector collapse.

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