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Measurement in quantum mechanics and classical statistical mechanics

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It is shown that the uncertainties $(\Delta P)^2$ and $(\Delta Q)^2$ in momentum and position of a quantum particle can always be expressed as the sum of a classical term and a quantum term. For quantum states characterized by a product $\Delta P \Delta Q \gg h$ it is always possible to reduce the uncertainty in both P and Q by performing measurements of *both* of them with resolutions ΔP^0 and ΔQ^0 such that their product is of the order of h. These measurements do not bring into existence values of P and Q which were nonexisting before, as it is usually assumed, but merely restrict the region in phase space allowed for them. This analysis can be used to support the thesis that the old question of the state vector collapse can be solved within the framework of the formalism of quantum mechanics, since it arises from the irreversible character of the increase of knowledge.

1. The absence of a general consensus among physicists about the old question of the state vector collapse induced by measurement in quantum mechanics is, after more than sixty years, puzzling. The large majority accepts as a postulate the traditional assumption that two mutually incompatible laws rule the time evolution of a system's physical state in quantum mechanics: the continuous, deterministic and reversible Schrödinger evolution of the state vector which takes place while the system is not observed, and its discontinuous, random and irreversible change (reduction, or collapse) induced by an act of measurement [1].

The remaining minority is aware of the existence of a problem, but is divided on its solution. Some consider this state of affairs as utterly unsatisfactory, and deny that a common explanation of these two incompatible time evolutions is possible in the framework of quantum mechanics. Only by modifying the Schrödinger equation in such a way as to make it capable of describing the objective nature of macroscopic bodies such as a measuring apparatus, while maintaining at the same time its validity at the quantum level, is it possible, according to this view, to find a way out of this difficulty [2].

Others insist that quantum mechanics has already a built-in mechanism which ensures its correct transition to the realm of classical physics for values of the action very large compared to the Planck constant h, and therefore hold that the laws of quantum mechanics are sufficient to explain both kinds of evolutions. This claim is based on the circumstance that whenever a microscopic system S interacts with a macroscopic instrument M all the probabilistic predictions derived from the solution of the overall Schrödinger equation for the global system S+M coincide *for all practical purposes* with those given by the corresponding statistical ensemble with the same possible outcomes [3].

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The crucial issue of this debate concerns, as is well known, the meaning of a superposition of different states (which are eigenstates of a macroscopic variable $\boldsymbol{G}$) of a macroscopic body, and its physical difference with the corresponding incoherent mixture. Those who invoke a modification of quantum mechanics admit that such a superposition may be in practice equivalent to the mixture of its components, but emphasize that the two situations are conceptually radically different.

The superposition is in fact a pure state in which, according to the generally accepted interpretation of quantum mechanics, the macroscopic variable $\boldsymbol{G}$ does not have a definite value, but acquires one of its possible values G_k only at the moment in which it is observed. This is of course what happens to the celebrated Schrödinger cat, which is neither dead or alive until one looks at it. The mixture instead describes a statistical assembly in which a large number of copies of the macroscopic body are distributed at random, with definite probabilities, among the possible states, each one being characterized by a definite value of $\boldsymbol{G}$, given a priori. To distinguish the superposition from the mixture, the argument goes on, it is sufficient to find a variable $\boldsymbol{F}$ of the macroscopic body which does not commute with $\boldsymbol{G}$, and therefore has different expectation values in the two situations.

The conclusion of the followers of this view is therefore that quantum mechanics in its present form contradicts the belief that macroscopic objects have intrinsic properties independently of whether they are observed or not, and therefore is incompatible with the conception that reality exists outside the human mind.

The opponents to this view recognize of course the conceptual difference between a superposition of states and their incoherent mixture, but insist that no *macroscopic* variable connecting different components of the superposition can be physically realized, and consequently that there is no practical way of detecting the difference between a superposition and the corresponding mixture for all the available macroscopic bodies (cats included).

On the other hand they recognize the possibility of defining in principle variables strongly dependent on the microscopic structure of the macroscopic body (such as multiple correlations between its constituent atoms) which might detect such a difference. This leaves the possibility open that, should one be able to prevent the destruction of the phase relations between the components by means of some clever device, one might discover new macroscopic quantum phenomena which would reveal complementary properties of macroscopic objects under mutually exclusive experimental conditions. In fact there are now indications that phenomena of this kind, such as macroscopic quantum tunneling (MQT) and macroscopic quantum coherence (MQC), may occur in nature [4].

The purpose of the present note is to provide an argument in favour of the thesis that quantum mechanics is not incompatible with the attribution of observer independent properties to macroscopic objects. The argument is based on a recent analysis of the properties of a representation of quantum mechanical states in phase space in terms of coherent states [5]. We show in fact that when a state is a superposition of macroscopically different states it is possible to give an objective meaning to the concept of "approximate" localization *in phase space* of the variables which describe the macroscopic properties of a quantum system, without preventing variables which probe the wavelike aspects of the state from showing the typical nonlocal behaviour of quantum mechanics. This behaviour allows a smooth transition from quantum mechanics to the domain of classical statistical mechanics in which both locality and realism are satisfied.

2. We start by considering the simplest quantum system defined by a space variable Q and its conjugate momentum P, which satisfy the usual commutation relation $[Q, P]=\mathrm{i}h$. We have shown in ref. [5] that it is useful to introduce creation and destruction operators,

$$A^\dagger=(2h)^{-1/2}(Q-\mathrm{i}P)\ ,$$

$$A=(2h)^{-1/2}(Q+\mathrm{i}P)\ , \qquad (1)$$

acting on a fiducial state $|0\rangle$ defined by $A|0\rangle=0$. We define the coherent state $|p, q\rangle$ as eigenstates of A with eigenvalue

$$a=(2h)^{-1/2}(q+\mathrm{i}p)\ . \qquad (2)$$

Then we have

$$|p,q\rangle=\exp[-(p^2+q^2)/4h]\times\sum_0^\infty[(2h)^{-1/2}(q+ip)]^n(n!)^{-1/2}|n\rangle\,,\qquad(3)$$

with the states $|n\rangle$ defined as usual by

$$|n\rangle=(n!)^{-1/2}(A^\dagger)^n|0\rangle\,.\qquad(4)$$

We will now discuss the statistical properties of functions of P and Q in the representation defined by (3). We first remark that for a sum of *anti*-ordered products of A and $A^\dagger$ with all the operators A on the left and all the operators $A^\dagger$ on the right one has

$$\langle\Omega|A^nA^{\dagger m}|\Omega\rangle=(2\pi h)^{-1}\int dp\,dq\,\langle\Omega|A^n|p,q\rangle\langle p,q|A^{\dagger m}|\Omega\rangle=\int dp\,dq\,\rho_t(p,q)a^na^{*m}\,,\qquad(5)$$

where ρ_t is a probability density given by

$$\rho_t(p,q)=\langle\Omega|p,q\rangle\langle p,q|\Omega\rangle\,.\qquad(6)$$

Therefore, since any polynomial operator $f(P,Q)$ can be written in the form

$$f(P,Q)=\sum_{nm}c(n,m)A^nA^{\dagger m}\,,\qquad(7)$$

we also have

$$\langle\Omega|f(P,Q)|\Omega\rangle=\sum_{nm}c(n,m)\int dp\,dq\,\rho_t(p,q)a^na^{*m}\equiv\int dp\,dq\,\rho_t(p,q)\varphi(p,q)\,.\qquad(8)$$

The function $\varphi(P,Q)$ differs from the classical counterpart $f(p,q)$ of the operator function $f(P,Q)$ only by terms of order h arising from the reordering implied in definition (7).

We have found [5] that, to any state $|\Omega\rangle$ it is possible to associate a positive probability density $\rho_t(p,q)$ (satisfying an evolution equation which reduces to the Liouville equation in the limit $h\to0$) and to any polynomial operator $f(P,Q)$ a c-number function $\varphi(P,Q)$ such that (8) holds. This result reminds of the analogous formulation of quantum mechanics in phase space proposed by Wigner, the only difference being that the Wigner function representing the state is not positive definite.

The distribution function $\rho_t(p,q)$ is not yet the limiting classical probability distribution in phase space corresponding to the quantum density matrix $|\Omega\rangle\langle\Omega|$ because it still depends on h. Its limit for $h\to0$, however,

$$\mathscr{P}(p,q)=\lim_{h\to0}\rho_t(p,q)\,,\qquad(9)$$

is indeed the required phase space probability distribution.

Therefore the expectation value of a quantum operator $f(P,Q)$ in any quantum state $|\Omega\rangle$ can always be expressed as the sum of an explicit classical term plus quantum corrections of order h:

$$\langle\Omega|f(P,Q)|\Omega\rangle=\int dp\,dq\,\mathscr{P}(p,q)f(p,q)+O(h)\equiv\underline{f}_{cl}+\underline{f}_q.\qquad(10)$$

The function $\mathscr{P}(p,q)$ is the classical probability distribution in phase space of the statistical mechanical ensemble corresponding to the density matrix $|\Omega\rangle\langle\Omega|$. The terms $O(h)$ are quantum corrections which vanish as $h\to0$.

We start by considering the statistical properties of the variables P and Q. For the mean square fluctuations $(\Delta P)^2$ and $(\Delta Q)^2$ of P and Q from their mean values, we may now write

$$(\Delta P)^2=\Delta p_{cl}^2+\Delta p_q^2\,,\qquad(11)$$

$$(\Delta Q)^2=\Delta q_{cl}^2+\Delta q_q^2\,,\qquad(12)$$

with

$$\Delta p_{cl}^2=\int dp\,dq\,(p-\underline{p})^2\mathscr{P}(p,q)\,,\quad \underline{p}=\int dp\,dq\,p\mathscr{P}(p,q)\,,\qquad(13)$$

$$\Delta q_{cl}^2=\int dp\,dq\,(q-\underline{q})^2\mathscr{P}(p,q)\,,\quad \underline{q}=\int dp\,dq\,q\mathscr{P}(p,q)\,.\qquad(14)$$

These results provide an extension to the case of the motion in an attractive potential $V(Q)$ regular at the origin of similar expressions found for free wave packets [6]. They are particularly relevant to the discussion of the meaning of the uncertainty principle and of all its consequences.

Equations (11), (12) allow one to make an important distinction between quantum states with both the quantities Δp^2_{cl}, $\Delta q^2_{cl} \gg h$ (in our units both p and q have dimensions $h^{1/2}$) and all the others. A simple example of a state of the first class is

$$\langle x|\Omega\rangle = (8\pi a^2)^{-1/4} \exp(-x^2/4a^2) \times [\exp(\mathrm{i}p_0 x/h) + \exp(-\mathrm{i}p_0 x/h)] , \qquad (15)$$

which gives

$$\mathscr{P}(p, q) = (8\pi a^2)^{-1/2} \exp(-q^2/2a^2) \times [\delta(p-p_0) + \delta(p+p_0)] \qquad (16)$$

and consequently

$$\Delta p^2_{cl} = p_0^2 \gg h , \quad \Delta q^2_{cl} = a^2 \gg h . \qquad (17)$$

A state of the first class therefore is practically equivalent to a phase space distribution of classical statistical mechanics because, since $\Delta P \Delta Q \approx \Delta p_{cl} \Delta q_{cl}$, uncertainties of quantum origin are negligible. States of the first class at $t=0$ will always belong to the first class because the classical uncertainties can only increase with time.

In the second class we find states with at least one of the uncertainties ΔP, ΔQ of the order h. This does not mean that $\Delta P \Delta Q$ is always $\approx h$. It means however that this product vanishes in the limit $h \to 0$. The states with $\Delta P \Delta Q > h$ have statistical properties with both a classical and a quantum component, while the statistical properties of the minimum uncertainty states are of purely quantum origin. An example of this kind is obtained by setting $p_0 = 0$ in eq. (15). One has

$$\langle p, q|\Omega\rangle\langle\Omega|p, q\rangle = N \exp[-q^2/(h+2a^2)] \times \exp[-2p^2 a^2/h(h+2a^2)] , \qquad (18)$$

which gives

$$\Delta p^2_{cl} = 0 , \quad \Delta q^2_{cl} = a^2 ,$$

$$\Delta q^2_{q} = 0 , \quad \Delta p^2_{q} = \frac{h^2}{4a^2} . \qquad (19)$$

This state is therefore a minimum uncertainty state with $\Delta P \Delta Q = \frac{1}{2}h$ even if the energy expectation value of the energy $\langle W \rangle \approx \frac{1}{2}a^2$ may be classically large. On the other hand if one takes a superposition of two Gaussians in coordinate space centered at $\pm x_0$ one gets $\Delta P \Delta Q = x_0 h/2a$ which for $x_0 \gg a$ may be $\gg \frac{1}{2}h$. In this case statistical properties of classical and quantum origin are intimately mixed. States of the second class will never become states of the first class even if the uncertainty product increases with time, because it will always remain proportional to h.

We propose therefore to assume that *the uncertainties in position Δq_{cl} and momentum Δp_{cl} for states of the first class have a classical statistical origin and should be accordingly interpreted in the same way as they are in classical statistical mechanics, namely as a consequence of the imperfect knowledge of the state of the system.* The statement that uncertainties whose product is of the order of h are the result of the irreducible fundamental quantum indeterminacy, which reflects the impossibility of simultaneous existence of position and momentum within a region in phase space of the order of h, maintains of course its full validity. This fundamental uncertainty is always present even when the classical uncertainty has been reduced. On the other hand for states of the first class one must be very careful not to ascribe to this quantum irreducible indefiniteness what is simply a statistical uncertainty due to lack of knowledge on the actual state of the system. In concise terms we propose to recognize that quantum interference effects take place only in phase space regions of order h.

To put it in another way, if a state is characterized by a product of uncertainties $\Delta P \Delta Q \gg h$, it is perfectly admissible to reduce the uncertainty in both P and Q by performing measurements of *both* of them with resolutions ΔP^0 and ΔQ^0 such that $\Delta P^0 \Delta Q^0 \approx \frac{1}{2}h$. This double measurement does *not* bring into existence values of P and Q which were nonexisting before, as is usually assumed, but merely performs the same function as of measurements in classical statistical mechanics, namely, as we shall see better in a moment, it restricts the region in phase space allowed for them. Of course this reduction of ignorance is not unlimited as it is in the classical case, because the quantum fundamental uncertainty prevents any further reduction of the product $\Delta P \Delta Q$ below $\frac{1}{2}h$.

We start to illustrate our statement by taking the example of a wave packet containing one particle (of the form (15)) impinging at $t=0$ on a diffraction analyzer which gives origin to two divergent wave packets of width $2a$ and momenta $\pm p_0$ (each one with uncertainty $\Delta p \approx h/2a$) emerging from it. After a time t the wave function of the particle in ordinary space is a superposition of two wave packets of width $\approx 2a$ separated by a distance $d \approx 2p_0 t$. In phase space the corresponding $\rho_t(p, q)$ is practically different from zero only in two disconnected regions, centered at $q_+ = p_0 t$, $p_+ = p_0$ and $q_- = -p_0 t$, $p_- = -p_0$, both with widths $\Delta q \approx a$, $\Delta p \approx h/2a$. When $d \gg a$ the uncertainty product is $\Delta P \Delta Q \approx \Delta p_{\text{cl}} \Delta q_{\text{cl}} = 2p_0 d \gg h$. Since the state belongs to the first class we can reduce the uncertainty by performing a position measurement (placing a counter along the path of one beam) with accuracy $\Delta q \approx a$. In this case there is no need to perform also a momentum measurement in order to reduce the uncertainty to the minimum, because the result $q = q_+$ (or $q = q_-$) yields automatically $p = p_+$ ($p = p_-$). Of course the results are both affected by the uncertainties $\Delta q \approx a$, $\Delta p \approx h/2a$, so that the minimum uncertainty product is attained.

Our statement that the particle is either in one or the other phase space region even before we put a counter along the path in order to detect it is in this case unambiguous, because the two possible subensembles corresponding to the minimum uncertainty product are identified before making the measurement. *Our interpretation reconciles therefore locality and realism: it is not the counter which materializes the particle in one of the two beams, but it is the particle which, being in one of the two beams, triggers the counter placed along its path* [#1].

Consider now the general case of a first class state whose support in phase space $\mathscr{R}$ has widths Δp_{cl} and Δq_{cl}. We perform *both* measurements of P and Q with resolutions $\Delta P^0 < \Delta p_{\text{cl}}$ and $\Delta Q^0 < \Delta q_{\text{cl}}$ such that $\Delta P^0 \Delta Q^0 \approx \frac{1}{2} h$. We stress that the two above limitations imply also that $\Delta P^0 > h/2\Delta q_{\text{cl}}$ and $\Delta Q^0 > h/2\Delta p_{\text{cl}}$. This entails that, if the results of the measurements are respectively $\underline{p}$ and $\underline{q}$, the a priori available classical region $\mathscr{R}$ reduces to the region spanned by all the possible minimum uncertainty wave packets centered in $\underline{q}$, $\underline{p}$ inside the boundaries $\underline{q} \pm \Delta Q^0$ and $\underline{p} \pm h/2\Delta Q^0$, with ΔQ^0 varying between its lower and upper limits. Also in the general case, therefore, we have an unambiguous reduction of our ignorance as a result of the joint measurements of P and Q.

This means therefore that there are two kinds of measurements. The first one reduces a wave packet with uncertainty $\Delta P \Delta Q \gg h$ into a wave packet with uncertainty $\Delta P^0 \Delta Q^0 \approx \frac{1}{2} h$. This measurement is irreversible because our knowledge changes irreversibly. It implies, exactly as it does in classical mechanics, the measurement of both P and Q, the only difference being that now the resolutions ΔP^0 and ΔQ^0 must satisfy the minimum uncertainty principle. The wave function changes because our knowledge of the statistical properties of the system is changed. This measurement eliminates the "empty waves" of a superposition because they are not physical: they only represent our ignorance before the measurement.

The second kind of measurement is a measurement of *either P or Q* performed on a minimum uncertainty wave packet. It corresponds to a change of the individual particle's physical state from a state with uncertainties $\Delta P'$ and $\Delta Q'$ to another state with uncertainties $\Delta P''$ and $\Delta Q''$, both products $\Delta P' \Delta Q'$ and $\Delta P'' \Delta Q''$ being equal to $\frac{1}{2} h$. There is clearly no reduction in this case, because there is no change in the information we have on the properties of the individual system: what we gain in the definition of Q (if $\Delta Q'' < \Delta Q'$) we lose in the definition of P ($\Delta P'' > \Delta P'$) and vice versa.

The main objection to this interpretation of the uncertainty relations is that even for states of the first class there may be quantities represented by functions $f(P, Q)$ of P and Q such that $\underline{f}_{\text{cl}}$ is much smaller than $\underline{f}_{\text{q}}$. This would mean that for this quantity the state is not almost classical as it appears from the behaviour of the variables P and Q. To give a simple example one can take for the state (15) a function of the form

$$f(Q) = \cos(2p_0 Q/b) . \tag{20}$$

It is easily found that one obtains in this case

$$\underline{f}_{\text{cl}} = \exp(-a^2 p_0^2 / b^2) ,$$

$$\underline{f}_{\text{q}} = \exp[-a^2 p_0^2 (b-h)^2 / h^2 b^2] . \tag{21}$$

[#1] We know that a common objection to this statement is that apparently it does not explain why the two beams interfere when in the absence of a counter they are brought together again. However when this happens the state is no longer a first class state, and therefore the uncertainty in position and momentum is again of quantum origin.

As long as $b \gg h$ one has, as expected, $f_{cl} \gg f_q$. When $b \approx h$ one has instead

$$f_{cl} = \exp(-a^2 p_0^2/b^2) \approx \exp(-a^2 p_0^2/h^2) ,$$
$$f_q \approx 1 , \tag{22}$$

namely $f_q \gg f_{cl}$.

This, however, is not a valid objection, because when $b \approx h$ the quantity $f(Q)$ is actually a quantum variable, since it depends crucially on h. It is therefore a quantity which probes the quantum mechanical details of the state and one should not therefore expect it to behave as a classical quantity. The important thing is that the quantum properties remain confined at the quantum level and do not penetrate into the classical level. This is indeed what the distinction between first and second class states ensures.

3. We have up to now dealt with the classical limit of quantum states for simple microscopic systems of one or a few degrees of freedom. These arguments may be however easily extended, in order to cope with the main issue of the debate on the nature of the collapse of the state vector in the course of the interaction between a microsystem and a measuring instrument, to the case of a macroscopic body M with N degrees of freedom made of a large number of microsystems.

In this case we have to distinguish between collective variables such as the center of mass coordinate Q and its conjugate momentum P, and the $N-1$ variables of the internal degrees of freedom. It is easy to show, at least for simple models of interaction between the individual microsystems [7], that the collective states depend only on h/N and therefore that the limit $N \to \infty$ implies the vanishing of Δq_q^2 and Δp_q^2. Collective states are therefore always first class states. For any macroscopic physical system N will be finite but $\gg 1$. This means that Δq_q^2 and Δp_q^2 will always be negligible compared to Δq_{cl}^2 and Δp_{cl}^2 respectively. Therefore, if we represent a state $|\Omega\rangle$ of M by $\langle p, q; q_i | \Omega\rangle$ the expectation value of a collective variable $f(P, Q)$ (independent of h) will be given, after integration on the internal variables q_i, by

$$\langle \Omega | f(P, Q) | \Omega \rangle = \int \mathrm{d}p\, \mathrm{d}q\, \mathscr{P}(p, q) f(p, q) + \mathrm{O}(1/N) , \tag{23}$$

with

$$\mathscr{P}(p, q) = \lim_{N\to\infty} (2\pi h)^{-1} \times \int \langle p, q; q_i | \Omega\rangle \langle \Omega | p, q; q_i\rangle \, \mathrm{d}q_i . \tag{24}$$

$\mathscr{P}(p, q)$ is the classical statistical distribution function in phase space corresponding to the quantum state $|\Omega\rangle$. It is now clear from our previous discussiôn that, since the minimum uncertainty quantum contributions coming from the terms $\mathrm{O}(1/N)$ are negligible, the averaging implied in (23) is of statistical nature, namely, it is a consequence of our ignorance about the actual state of M in phase space which, however, does have practically definite values of p, q (within the fundamental uncertainty $\Delta p \Delta q \approx \frac{1}{2} h$) independently of our knowledge. This means that the quantum irreducible randomness at the quantum level does not generate, as Schrödinger feared and many physicists still believe, essential indefiniteness of physical quantities at the classical level [#2].

In line with the preceding arguments we can now conclude that the collapse of a microsystem's state vector as a consequence of the measurement of one of its variables by means of an instrument is no longer an unsolved puzzle. It is easy to see why.

With standard notation let us denote the state of the system S+M, after the interaction which has established a one-to-one correspondence between the eigenstates ϕ_k of the variable $\boldsymbol{g}$ (with eigenvalues g_k) of S and the eigenstates $|k\rangle$ of the variable $\boldsymbol{G}$ (with eigenvalues G_k) of M, as follows,

$$|\Omega\rangle = \sum_k c_k \phi_k |k\rangle . \tag{25}$$

[#2] This statement may sound too strong in view of the evidence for the quantum phenomena quoted in ref. [4] involving macroscopic quantities such as current or magnetic flux. It should be kept in mind however that, even if the existence of such phenomena is confirmed, the action involved (e.g. height of the potential barrier times the period of oscillation in the case of MQC) is always of order h.

Then the expectation value of a collective variable $f(P, Q)$ of the instrument will be given by

$$\langle \Omega | f(P,Q) | \Omega \rangle = \sum_k |c_k|^2 \times \int \mathrm{d}p\, \mathrm{d}q\, \mathscr{P}_k(p,q) f(p,q) + \mathrm{O}(1/N) , \tag{26}$$

where the functions $\mathscr{P}_k(p, q)$ are the set of phase space classical probability distributions which correspond to the disconnected phase space regions where $\boldsymbol{G}$ has the different values G_k. The average implied in (26) is therefore of purely statistical origin and reflects our ignorance on the actual state of the individual instrument M which, however, does have a definite value, say, G_j of $\boldsymbol{G}$ independently of our knowledge. The collapse of the state of S+M from the superposition (25) to the single state $\phi_j|j\rangle$ when an individual system of the statistical ensemble is chosen is therefore only a matter of reduction of ignorance, not a physical phenomenon in which M is involved.

In order to avoid confusions we want to summarize our argument by stressing that we are not only saying, as many people, including one of us, have done, that the density matrix W of the state $|\Omega\rangle$ reduces practically to the density matrix $\underline{W}$ of the corresponding mixture

$$\underline{W} = \sum_k |c_k|^2 \phi_k \phi_k^* |k\rangle\langle k| , \tag{27}$$

when the states $|k\rangle$ of M are macroscopically different. This still leaves open the question of the nature and the origin of the collapse from the sum in eq. (27) to a single term $\phi_j\phi_j^* |j\rangle\langle j|$ in an individual measurement. We are saying more, namely that, since the classical limit $\sum_k |c_k|^2\, \mathscr{P}_k(p, q)$ of the representation of the density matrix W in the phase space of M given by (26) coincides with the phase space probability distribution of classical statistical mechanics, the collapse to the single term $\mathscr{P}_j(p, q)$ has the same nature and origin of the same phenomenon in classical statistical mechanics. It represents therefore an irreversible reduction of our ignorance of the actual value G_j of $\boldsymbol{G}$ due to the act of looking at the instrument's pointer.

It seems therefore that, in order to give Schrödinger's cat a longtime due honoured burial, it is sufficient to interpret correctly the classical limit of quantum mechanics.

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