MACROSCOPIC QUANTUM COHERENCE AS A TEST OF QUANTUM MECHANICS

L. CHIATTI
Laboratorio di Fisica Medica, CRS IRE, Roma

M. CINI
Dip. di Fisica Università "La Sapienza", Roma

M. SERVA
Dip. di Matematica and INFN, L'Aquila

Abstract. The proposal of using Macroscopic Quantum Coherence in a SQUID as a test of the validity of Quantum Mechanics (QM) for macroscopic systems (Leggett A., 1980; Leggett A. and Garg A. 1985) is considered. We note that if only Macroscopic Realism (MR) is assumed but the requirement that the flux measurement is non invasive (NIM) is dropped, only the measurement of the charge would discriminate between QM and MR. This discrimination however depends critically on the experimental parameters. There is a threshold above which QM is consistent with MR but the measurement is invasive as in QM.

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1. Introduction

The main property of Quantum Mechanics (QM) is the superposition principle of states:

$$\psi = a_1\psi_1 + a_2\psi_2 \qquad (1)$$

There is no question about its validity for microscopic systems. One should ask however whether it is also valid for macroscopic bodies. In fact, if the system is in the state (1) one has to accept that the variable G with eigenstates ψ_1 and ψ_2 is essentially undetermined until it acquires a definite value g_1 or g_2 in the act of

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measurement (Schrödinger's cat). Since we are accustomed to believe in macrorealism, namely to assume that the variables of macroscopic bodies always have a definite value (even if unknown to us) it is impossible to reconcile the validity of QM with our everyday experience.

It is therefore extremely interesting to investigate whether the possibility exists of making a macroscopic system such that the coherence effects of the linear superposition (1) may be detected. One of the most promising devices in this perspective is provided by the flux oscillations in a SQUID (Superconducting Quatum Interference Device) [Leggett A. 1980, 1986,1987].

The SQUID, as is well known, is a superconducting ring interrupted by a Josephson junction (JJ). The magnetic flux Φ through the ring reverses its sign as the current (of the order of 10^{22} Cooper pairs) oscillates in the ring. The junction is a thin non conducting barrier allowing the leakeadge of a current i_J. The SQUID is caracterized by a capacitance C and an inductance L. We neglect the resistance for the present considerations. The basic equations of the SQUID are the following.

If we denote by

$$\psi = \rho^{1/2}\, e^{i\theta} \tag{2}$$

the common wave function of the Cooper pairs in the superconducting ring, the standard expression of the current **j**

$$\mathbf{j} = (\rho/m)[(h/2\pi)\, \mathrm{grad}\theta - q\mathbf{A}] \tag{3}$$

gives

$$(h/2\pi)(\theta_1 - \theta_2) = q \int \mathrm{rot}\mathbf{A}\, d\mathbf{S} = q\, \Phi \tag{4}$$

where $\theta_1 - \theta_2$ is the phase difference on the two sides of the junction, because $\mathbf{j} = 0$ inside the superconductor.

Furthermore the current i_J through JJ is (q=2e):

$$i_J = i_o \sin(\theta_1 - \theta_2) = i_o \sin(4\pi e\Phi/h) \tag{5}$$

where i_o is the critical current.

It is straightforward now to write down the classical energy of the SQUID:

$$H = (1/2)\, C\, V^2 + (1/2)\, L\, i^2 + \int V\, i_J\, dt \tag{6}$$

namely, introducing the variable Φ and the elementary fluxon $\Phi_o = h/2e$

$$H_c = (1/2)C(d\Phi/dt)^2 + (1/2L)(\Phi-\Phi_{ext})^2 - (\Phi_o i_o/2\pi)\cos(2\pi\Phi/\Phi_o) \qquad (7)$$

We notice that the momentum p_Φ conjugated to Φ is

$$p_\Phi = C\, d\Phi/dt = Q \qquad (8)$$

namely the charge Q stored in the SQUID.

We now transform the classical Hamiltonian (7) into a quantum Hamiltonian by replacing p_Φ with the operator $(h/i)\partial/\partial\Phi$. We thus have:

$$H_q = -\,((h/2\pi)^2/2C)\partial^2/\partial\Phi^2 + U(\Phi) \qquad (9)$$

with

$$U(\Phi) = (1/2L)(\Phi - \Phi_{ext})^2 - (\Phi_o i_o/2\pi)\cos(2\pi\Phi/\Phi_o) \qquad (10)$$

The external flux Φ_{ext} may be chosen equal to $\Phi_o/2$. Then U becomes symmetrical around $\Phi = \Phi_o/2$ for suitably chosen values of L, i_o and C. $U(\Phi)$ has the shape of a double well potential with the two minima Φ_+ , Φ_- such that

$$\Phi_+ + \Phi_- = \Phi_o \qquad 0 < \delta\Phi = \Phi_+ - \Phi_- < \Phi_o \qquad (11)$$

Near each minimum the potential is approximately an harmonic oscillator potential of frequency ω. Denoting by $\psi_\sigma(\Phi)$ (with $\sigma=\pm1$) the ground state wave functions centered respectively at Φ_+,Φ_-, the Schrödinger equation

$$H_q\Psi(\Phi,t) = i\,(h/2\pi)\,\partial\Psi(\Phi,t)/\partial t \qquad (12)$$

admits solutions with initial value $\Psi(\Phi, t_o)= \psi_\sigma(\Phi)$ of the form

$$\Psi(\Phi,t) = \psi_\sigma(\Phi)\cos\Omega(t-t_o) - i\,\psi_{-\sigma}(\Phi)\sin\Omega(t-t_o) \qquad (13)$$

The frequency Ω may be calculated in terms of the potential parameters and is generally << ω. The probabilities of finding at time t the flux Φ in the states $\psi_\sigma(\Phi)$, $\psi_{-\sigma}(\Phi)$ are therefore

$$P(\sigma,t;\ \sigma,t_o) = \cos^2\Omega(t-t_o) \qquad P(-\sigma,t;\ \sigma,t_o) = \sin^2\Omega(t-t_o) \qquad (14)$$

2. Macroscopic Quantum Coherence as a test of Quantum Mechanics for macroscopic systems

A. Leggett and A. Garg have proposed [Leggett A., Garg A. 1985] to use the transitions of the magnetic flux trapped in a SQUID as a test of the validity of QM for macroscopic systems. More precisely they argue that the predictions of QM are incompatible with the following two assumptions which should characterize the behaviour of a macroscopic system:

(a) Macroscopic Realism (MR), namely that a macroscopic system will always be in either one or the other of two macroscopically distinct states;
(b) Non Invasive Measurability (NIM), namely that it is possible to determine the state of the system with arbitrarily small perturbation.

The proof goes as follows. Denote by σ_i (±1) the value of Φ measured at time t_i. Then from (a) one derives

$$\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_4 - \sigma_1\sigma_4 \leq 2 \qquad (15)$$

By averaging on a statistical ensemble and introducing the two times correlations K_{ij}

$$K_{ij} = \langle \sigma_i\sigma_j \rangle \qquad (16)$$

one gets

$$K_{12} + K_{23} + K_{34} - K_{14} \leq 2 \qquad (17)$$

Assumption (b) comes into play because if one wants to compare QM with MR one must assume that, by making flux measurements at t_1, t_2 or t_2, t_3 or t_3, t_4 or t_1, t_4 on *different* ensembles (since the QM predictions for K_{ij} are only valid if the evolution between t_i and t_j is not disturbed by intermediate measurements) the results are the same *as if* the measurements would be performed on a unique ensemble at all times.

On the other hand eq. (14) shows that QM gives:

$$K_{ij} = \Sigma_\sigma\ \sigma_i\sigma_j P(\sigma_i t_i;\ \sigma_j t_j) = \cos^2\Omega(t_i-t_j) - \sin^2\Omega(t_i-t_j) = \cos 2\Omega(t_i-t_j) \qquad (18)$$

This expression violates the Bell type inequality (17) (e.g. if we take t_2-t_1 = t_3-t_2 = t_4-t_3 = $\pi/8\Omega$ the l.h.s. is $2\sqrt{2}$). QM is therefore incompatible with MR+NIM.

3. Macroscopic Quantum Coherence and MR

It may be however interesting to investigate whether a separate test of MR would be possible without assuming NIM. An experimental proposal to test NIM independently has in fact been recently proposed [Cosmelli et al. 1993] and is currently in preparation. In fact, NIM is independent of MR and one may well conceive a theory in which the system will always have either one or the other of the two values of the flux, but the time evolution of the relevant probabilities will start again from the new initial condition corresponding to the measured value of Φ whenever a measurement is performed, as in QM.

A macrorealistic model of this type is obtained [Cini M. and Serva M. 1990, 1992] by describing the system by means of the density matrix

$$\underline{W}(t) = \psi_\sigma(\Phi)\, \psi^*_\sigma(\Phi) \cos^2 \Omega(t\text{-}t_o) + \psi_{-\sigma}(\Phi)\, \psi^*_{-\sigma}(\Phi) \sin^2 \Omega(t\text{-}t_o) \qquad (19)$$

obtained from the density matrix W(t) of the pure state (13) by dropping the off-diagonal terms. The density matrix $\underline{W}(t)$ describes in fact a statistical mixture in which the flux is *either* in the state $\psi_\sigma(\Phi)$ *or* in the state $\psi_{-\sigma}(\Phi)$, representing the limit of the quantum density matrix when these states are macroscopically different [Cini M. 1983]. In fact when the distance between the two wells becomes macroscopically large, the overlap between $\psi_\sigma(\Phi)$ and $\psi_{-\sigma}(\Phi)$ vanishes exponentially. The same thing happens for the off-diagonal contributions to the mean value of any observable with a classical limit.

It is then clear that the predictions of the MR model for the flux measurements are identical to those of QM because both $\underline{W}(t)$ and W(t) give the same probabilities (20) of finding the flux in either the state $\psi_\sigma(\Phi)$ or $\psi_{-\sigma}(\Phi)$ at time t. It is therefore also clear that no experiment in which only the flux is measured can distinguish between this MR model and QM.

Only if one measures the variable Q conjugated to Φ can one test therefore the validity of the superposition principle for the macroscopic flux coherent oscillations in the double well. In fact, since

$$Q = (h/2\pi i)\, \partial/\partial\Phi \qquad (20)$$

one obtains, for the wave function $\Theta(Q, t)$ in Q-space

$$\Theta_\sigma(Q, t) = (h)^{-1/2} \int \Psi_\sigma(\Phi, t) \exp(-2\pi i Q\Phi/h)\, d\Phi \qquad (21)$$

By using (13) and taking into account that the ground state oscillator wave functions $\psi_\sigma(\Phi)$ are given by

$$\psi_{\pm1}(\Phi) = (\beta/\pi)^{1/4} \exp[-\beta(\Phi-\Phi_\pm)^2/2], \qquad \beta= 2\pi C\omega/h \qquad (22)$$

we obtain the probability $P_\sigma(Q, t) = |\Theta_\sigma(Q, t)|^2$ of finding the value Q of the charge at time t

$$P_\sigma(Q, t) = 2(\pi/\beta h^2)^{1/2}\exp[-4\pi^2Q^2/\beta h^2]\{1+\sigma\sin 2\Omega(t-t_o) \sin[2\pi Q\delta\Phi/h]\} \qquad (23)$$

The oscillating interference term is characteristic of QM. In the MR model (19) this term is instead missing. It is therefore possible, at least in principle, to discriminate between QM and MR by measuring the variable Q. The easiest thing to do might be to determine the sign of Q. Then the probability $P_\sigma(Q>0)$ is given by:

$$P_\sigma(Q>0, t) = (1/2)\{1 + \sigma \exp[-\beta\, (\delta\Phi)^2/4] \sin 2\Omega(t-t_o)\} \qquad (24)$$

The coefficient of the oscillating term, however, depends critically on the actual values of the experimental parameters. If one takes the values $C\approx2.10^{-13}$F, $\omega\approx3.10^9$ Hz, $\delta\Phi\approx\Phi_o/2$ from the available experimental proposals [Tesche C.D. 1987, Cosmelli et al. 1993], the absolute value of the argument of the exponential is of the order of one. This means that an increase of these parameters of a factor of some units is sufficient to reduce drastically the visibility of the quantum interference effect. If this happens QM is consistent with MR because there is no observable difference between the quantum mechanical density matrix W(t) and the density matrix $\underline{W}$(t) of the corresponding mixture. However, in this region, the measurement is still invasive as in QM.

It will therefore be of great interest to investigate experimentally the transition between the quantum and the classical region.

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