Economics Letters 65 (1999) 275–279

economics letters

www.elsevier.com/locate/econbase

Multiscale behaviour of volatility autocorrelations in a financial market

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Received 2 November 1998; accepted 12 May 1999

Abstract

We perform a scaling analysis on NYSE daily returns. We show that volatility correlations are power-laws on a time range from one day to one year and, more important, that the exponent is not unique, consistently with a multiscale behaviour.

Keywords: Daily returns; Volatility; Correlations; Multiscaling

JEL classification: C10

It is well known that stock market returns are uncorrelated on lags larger than a single day, in agreement with the hypothesis of efficient market. On the contrary, absolute returns have memory for longer times; this phenomenon is known in financial literature as *clustering of volatility*. In ARCH-GARCH models (Engle, 1982; Jorion, 1995; Andersen and Bollerslev, 1998), volatility memory is longer than a single time step but it decays exponentially. Since empirical evidence is for hyperbolic correlations (Taylor, 1986; Ding et al., 1993; Baillie and Bollerslev, 1994; Crato and de Lima, 1994; Baillie, 1996; Pagan, 1996), GARCH models have been extended in order to take into account this phenomenology (Ding et al., 1993; Harvey, 1993; De Lima et al., 1994; Baillie et al., 1996).

In this paper, we perform a scaling analysis of the standard deviation of a new class of observables, the *generalized cumulative absolute returns*. This analysis clearly shows that volatility correlations are power-laws on a time range from one day to one year and, more important, that the exponent is not unique. This kind of multiscale behaviour is known to be relevant in the theory of dynamical systems,

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0165-1765/99/$ – see front matter
PII: S0165-1765(99)00159-7

of fully developed turbulence and in the statistical mechanics of disordered systems (see Paladin and Vulpiani, 1987, for a review) while it is a new concept for financial modeling.

We consider the daily New York Stock Exchange (NYSE) index, from January 1966 to June 1998, for a total of $N = 8180$ working days. The quantity we consider is the (de-meaned) daily return, defined as

$$r_t = \log \frac{S_{t+1}}{S_t} - \left\langle \log \frac{S_{t+1}}{S_t} \right\rangle \tag{1}$$

where S_t is the index value at time t ranging from 1 to N, and $\langle \cdot \rangle$ is the average over the whole sequence. The underlying daily volatility σ_t is not directly observable, but it is indirectly defined by $r_t = \sigma_t \eta_t$. It is assumed that the η_t are identically distributed random variables with vanishing average and unitary variance. The usual choice for the distribution of the η_t is the normal Gaussian. The observables directly related to the volatilities are the absolute returns $|r_t|$.

As pointed out by several authors (Mandelbrot, 1963; Clark, 1973; Mantegna and Stanley, 1995), the distribution of returns is leptokurtic. In Mandelbrot (1963), it was firstly proposed a symmetric Lévy stable distribution and more recently in Mantegna and Stanley (1995) it has provided strong evidence for this fact. More precisely, in Mantegna and Stanley (1995) it is shown that the distribution is Lévy stable for high frequency returns except for tails, which are approximately

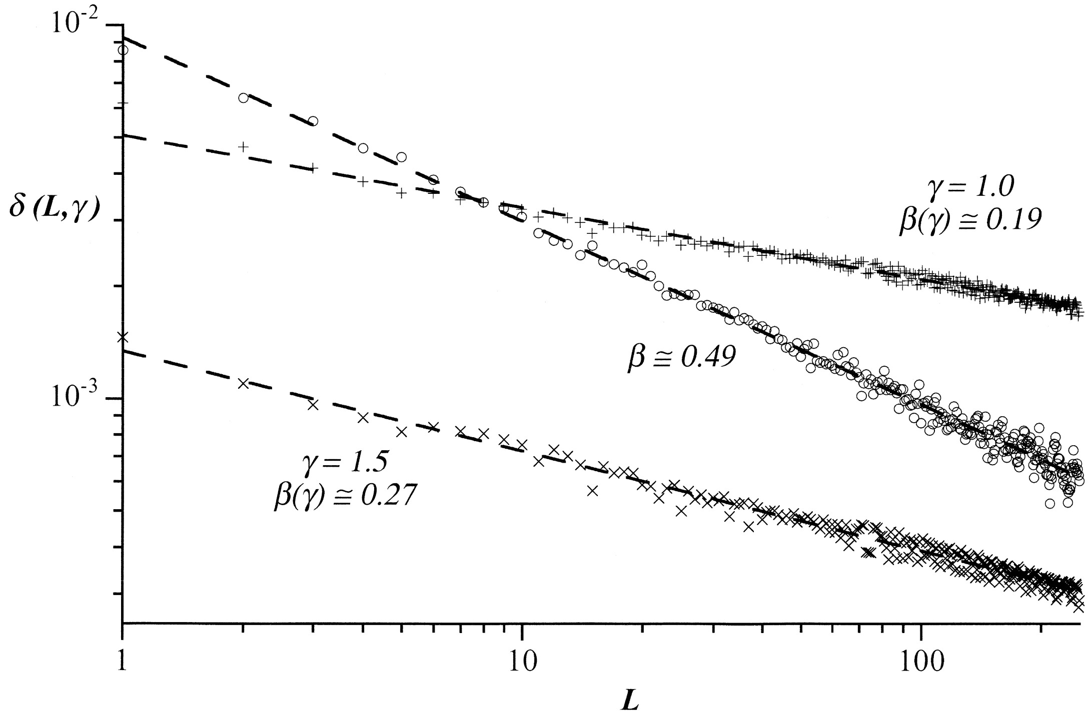

Fig. 1. Standard deviation $\delta(L,\gamma)$ of the generalized cumulative absolute returns (2) as a function of L on log–log scales for $\gamma = 1$ (crosses) and $\gamma = 1.5$ (slanting crosses), compared with the standard deviation of the cumulative returns (circles). The exponents of the best fit straight lines (dashed lines) are, respectively, $\beta(1) \simeq 0.19$, $\beta(1.5) \simeq 0.27$ and $\beta \simeq 0.49$.

exponential. The estimation is that the shape of a Gaussian is recovered only on longer scales, typically one month.

Our analysis is on low frequency data, and first of all we want to verify that anomalous Lévy scaling is not effective in a range of time from one day to one year. We consider the *cumulative returns* $\phi_t(L)$, defined as the sum of L successive returns $r_t, \ldots, r_{t+L-1}$, divided by L. Using NYSE data one can define N/L not overlapping variables of this type and compute the standard deviation $\sigma(L)$. The standard deviation is independently computed for L ranging from 1 to 250 (one year). Larger value of L would imply insufficient statistics. Assuming that r_t are uncorrelated (or short range correlated), it follows that $\sigma(L)$ has a power-law behaviour with exponent 0.5 for large L, i.e. $\sigma(L) \sim L^{-\beta}$ with $\beta = 0.5$. The exponent for the NYSE index turns out to be about 0.49 (see Fig. 1 and also see Mantegna and Stanley (1996)), according to the hypothesis of uncorrelated returns. This value of the exponent also ensures that Lévy scaling is not effective in this range of time.

Let us introduce the *generalized cumulative absolute returns* defined as

$$\chi_t(L,\gamma) = \frac{1}{L} \sum_{i=0}^{L-1} |r_{t+i}|^{\gamma} \tag{2}$$

where γ is a real exponent and again, these quantities are not overlapping. If the $|r_t|^{\gamma}$ are uncorrelated, one should find that the standard deviation $\delta(L,\gamma)$ has a power-law behaviour with exponent 0.5.

On the contrary, a power-law autocorrelation function with exponent $\alpha(\gamma) \leq 1$, $\langle |r_t|^{\gamma} |r_{t+L}|^{\gamma} \rangle -$

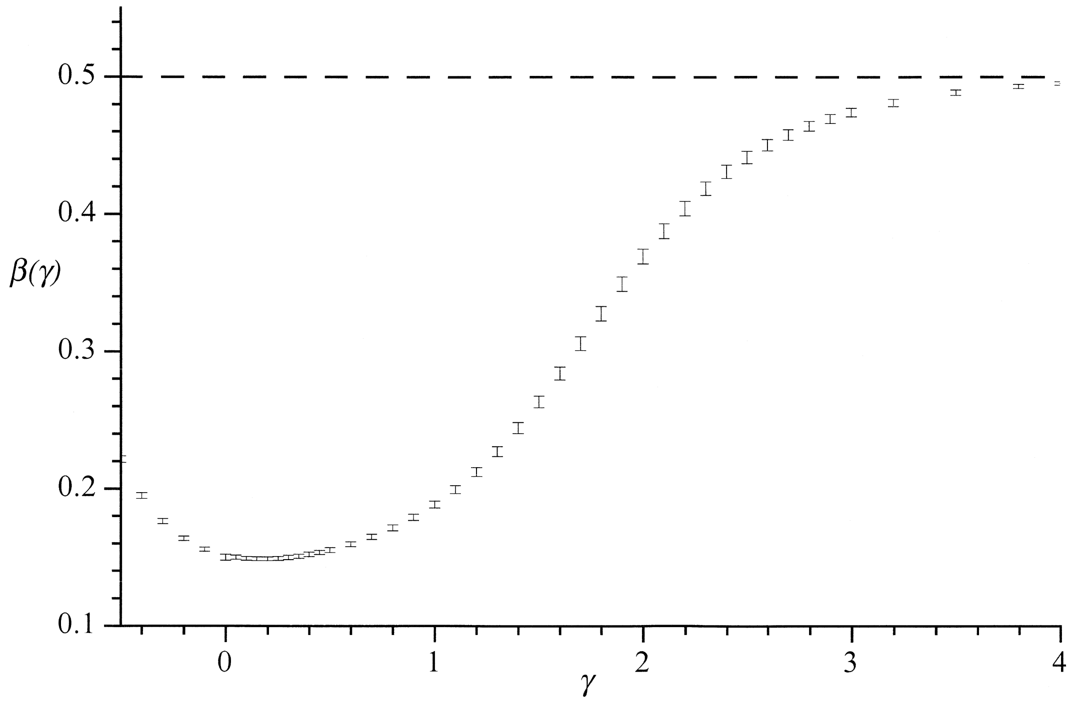

Fig. 2. Scaling exponent $\beta(\gamma)$ of the standard deviation $\delta(L,\gamma)$, where the bars represent the errors over the best fits. An anomalous scaling ($\beta < 0.5$) is shown in the range $-0.5 < \gamma < +4$.

$\langle|r_t|^\gamma\rangle\langle|r_{t+L}|^\gamma\rangle \sim L^{-\alpha(\gamma)}$, would imply that $\delta(L,\gamma)$ is a power-law with exponent $\beta(\gamma)=\alpha(\gamma)/2$. For autocorrelations with exponent $\alpha(\gamma)\geq 1$ we would not detect anomalous scaling for the standard deviation ($\beta(\gamma)=0.5$).

Our numerical analysis on the NYSE index shows very sharply that $\delta(L,\gamma)$ has an anomalous power-law behaviour in the range from one day to one year ($L=250$). For example, for $\gamma=1$ we find $\beta(1)\simeq 0.19$, while for $\gamma=1.5$, $\beta(1.5)\simeq 0.27$ (see Fig. 1). For larger L the statistics becomes insufficient.

The crucial result is that $\beta(\gamma)$ is a not constant function of γ in the range $-0.5<\gamma<+4$ (see Fig. 2), showing the presence of different scales. The interpretation is that different γ select different typical fluctuation sizes, any of them being power-law correlated with a different exponent.

The longest correlation is for $\gamma=0.15$ ($\beta(0.15)\simeq 0.15$). The case $\gamma=0$ corresponds to cumulative logarithm of absolute returns.

In the region $\gamma \gtrsim 4$ the averages are dominated by only few events, corresponding to very large returns, and, therefore, the statistics becomes insufficient.

The anomalous power-law scaling can be directly tested against the plot of autocorrelations. For instance, the autocorrelations of r_t and of $|r_t|$ are plotted in Fig. 3 as a function of the correlation length L. Notice that the full line, which is in a good agreement with the data, is not a best fit but it is a power-law whose exponent $2\beta(1)\simeq 0.38$ is obtained by the previous scaling analysis of the standard deviation. The autocorrelations for the return, as expected, vanish except for the first step ($L=1$).

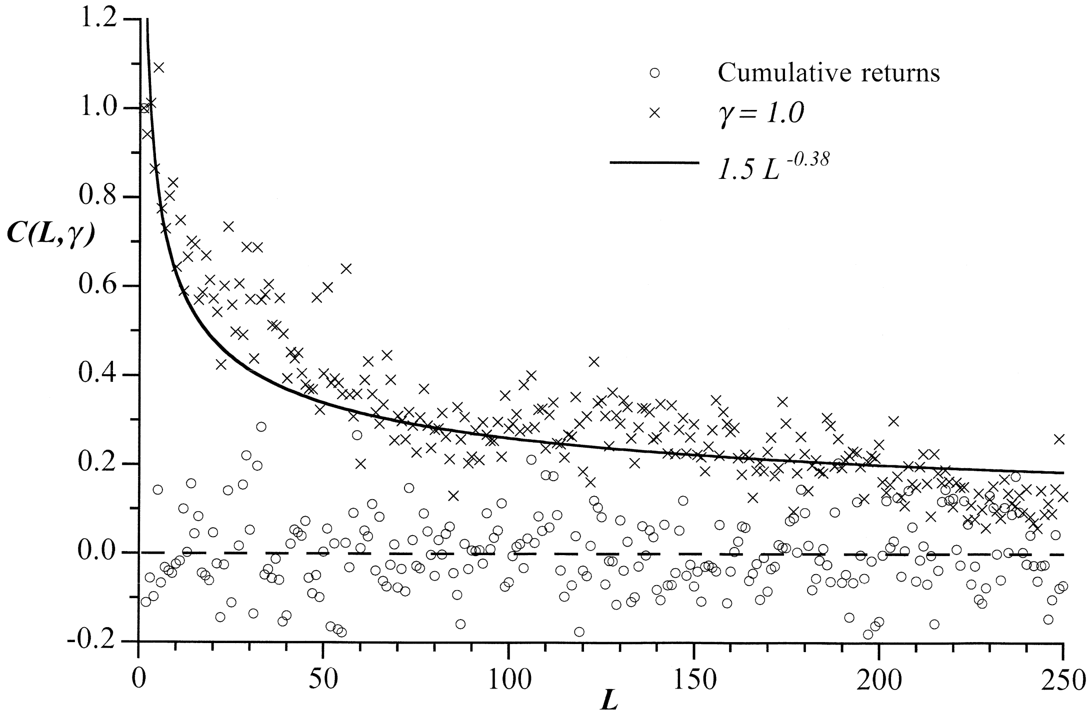

Fig. 3. Autocorrelation function of $|r_t|$ (crosses) as a function of the correlation length L, compared with the autocorrelation function of r_t (circles). The data are in a good agreement with a power-law with exponent $2\beta(1)\simeq 0.38$ in the first case, and absence of correlations in the second. In both cases the scale is fixed by autocorrelations equal to 1 at $L=1$.

It should be also noticed that a direct analysis of the autocorrelations would not have provided an analogous clear evidence for multiscale power-law behaviour, since the data show a wide spread compatible with different scaling hypothesis.

In conclusion, we have analyzed the scaling behaviour of the generalized cumulative absolute returns, showing that it is consistent with power-law correlations with not unique index. We suggest that volatility models should take into account this multiscale phenomenology.

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