

Indeterminacy in foreign exchange markets

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Abstract

We discuss price variations distributions in foreign exchange markets, characterizing them both in calendar and business time frameworks. The price dynamics is found to be the result of two distinct processes, a multi-variance diffusion and an error process. The presence of the latter, which dominates at short time scales, leads to indeterminacy principle in finance. Furthermore, dynamics does not allow for a scheme based on independent probability distributions, since volatility exhibits a strong correlation even at the shortest time scales. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction and definition of time

The characterization of price dynamics in financial markets is an old but still puzzling problem. At the beginning of the century, Bachelier [1] proposed to consider price variations as independent realizations of identically Gaussian distributed variables, while in the 1963 Mandelbrot [2] introduced symmetric Lévy stable distributions. In the last years, Mantegna and Stanley [3] (and see also Refs. [4,5]) provided evidence that a Lévy stable process well reproduces the central part of high-frequency price variation distribution, while the tails are approximately exponential. The Bachelier's Gaussian shape is recovered only on longer time scales which are, typically, of order one month. A common point of [1–5] is that price dynamics is considered as the result

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of independent random variables. This kind of approach seems inadequate, since there is evidence of volatility correlations on long run [6–13].

Indeed, price distribution strongly depends on how one measures time flow. The choice of time index is twofold, calendar time and business time. Business time is the sequence of integers $n = 1, 2, 3, \dots$ which indexes successive established quotes. These integers correspond, respectively, to the calendar times t_1, t_2, \dots . Therefore, calendar time (which is monotonically increasing) is a stochastic process of business time. The relation can be inverted by considering n as a function of calendar time, i.e., $n = n_t$, but in this case the function is defined only on the sequence of quoting calendar times. The price dynamics, therefore, can be described with respect to business time by means of series S_n or with respect to the calendar time by means of $S_t \equiv S_{n_t}$. In the first cases quotes S are defined on all integers and lags are all equal, in the second, they are defined only on quoting times and lags are unequal.

The main result of this paper is that a price quote is due to two distinct independent stochastic processes: an *error* process superposed to an *underlying* price process. The latter evolves following calendar time, while the former is due to an erroneous evaluation of a market operator, and its natural frequency is marked by business time. The resulting price variation distribution is, therefore, the convolution of two distributions associated to these two distinct processes: the error process distribution, which does not change with time, and the distribution of the underlying process, which scales, on the contrary, with calendar time.

The error process produces always a gap between two consecutive price estimations even if they are almost contemporary. This phenomenology strongly reminds quantum mechanics, where a measurement result always has a minimum uncertainty as stated by the Heisenberg Principle. Following this comparison, one can state an *indeterminacy principle* in finance: a price is never given with a precision less than a natural constant for that market.

Another important fact which is due to this phenomenology is that two consecutive price variations cannot be considered fully independent random variables, but they exhibit a very peculiar anticorrelation as we will see. Moreover, we provide evidence that volatility is so strongly correlated that remains substantially constant inside the largest lag we consider, and therefore the usual Lévy stable scheme seems to be not appropriate.

In this paper we examine the high-frequency price variation distribution of three foreign exchange markets, the Deutsch Mark/US Dollar (DEM-USD) exchange in 1993 (1,472,140 quotes) and in 1998 (1,620,843 quotes), the Japanese Yen/US Dollar (JPY-USD) in 1993 (570,713 quotes), and the Japanese Yen/Deutsch Mark (JPY-DEM) in 1993 (158,878 quotes). The quotes represent the value of one US Dollar in Deutsch Marks and in Japanese Yens respectively, the DEM-USD and JPY-USD cases, while they represent the value of one Deutsch Mark in Japanese Yens in the DEM-JPY case. The price changes are given in pips, which indicate a DEM/10,000 in the DEM-USD case, and a Yen/100 in the JPY-USD and JPY-DEM cases. All the data sets analyzed in this work have been provided by Olsen & Associates.

We deal only with bid price, since we have found more spurious data in the ask price distributions due to wrong transcriptions. This is probably related to the fact that in recording process quotes are specified by bid price and the last two digits of the bid/ask spread. Nevertheless, all the results can be fully reproduced if one deals with ask price or average price.

The paper is organized as follows. In Section 2 it is shown that a price variation is given by the composition of two distinct stochastic processes and that the hypothesis of independent consecutive price variations is incorrect, due to the presence of both price anticorrelation and volatility correlation. In Section 3 the price variation distribution for a single business lag and for the minimum calendar lag (2 s) are computed. The latter turns out to be, basically, the error distribution. In Section 4 we provide evidence that the underlying price variation distribution at a generic calendar time lag is given by a symmetric Gaussian process whose standard deviation is itself a log-normal process. In Section 5 some final remarks are reported.

2. The indeterminacy principle in finance

The aim of this section is to show that the set of reported bid quotes is a realization of stochastic process which is the composition of two processes whose origin and meaning are very different. The first is the ordinary multiplicative stochastic process which determines the evolution of underlying price, while the second is a superposed stochastic noise which somehow accounts for the erroneous evaluation of the underlying price by the market operators.

In order to demonstrate our point it is better to consider the price evolution in business time.

Let us assume that there exists an unobserved underlying price \tilde{S}_n which accounts for the real relative value of two currencies and which follows the ordinary evolution rule

$$\tilde{S}_{n+1} = \tilde{S}_n + \tilde{R}_n, \quad (1)$$

where, as usual, the \tilde{R}_n are, at the lowest approximation, identically distributed variables which: (a) have vanishing average, (b) only depend on the stochastic calendar lag $\Delta t_n \equiv t_{n+1} - t_n$, (c) are uncorrelated, i.e., the average of the product $\tilde{R}_n \tilde{R}_m$ vanishes for $n \neq m$, (d) their typical size (standard deviation) is proportional to \tilde{S}_n . The last property is that it is necessary to ensure that the process is multiplicative on the large time scale. Note that we do not assume that the \tilde{R}_n are independent, and the reason for that will be clear at the end of this section.

Let us also assume that the observed price (which is the recorded quote) slightly differs from the underlying price because of an erroneous evaluation of the operator. The relation between the *underlying* and the *observed* price will be

$$S_n = \tilde{S}_n + E_n, \quad (2)$$

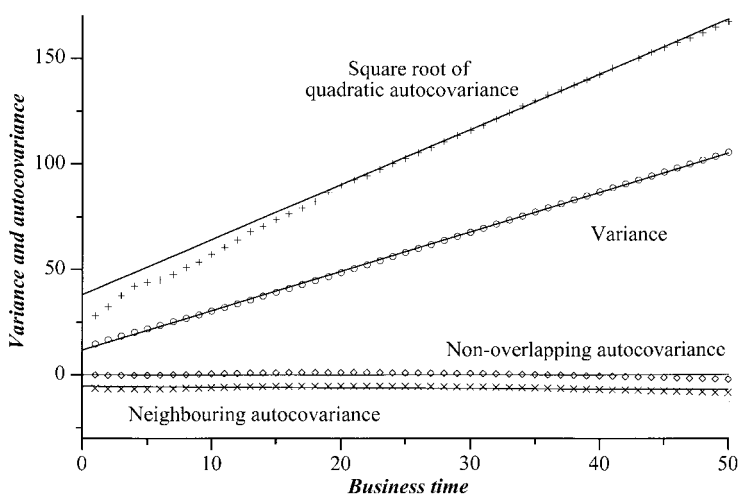


Fig. 1. DEM-USD 1998 exchange rate: variance (4) (circles) compared with a linear fit $2A + Bm$ with $A = 5.9 \pm 0.2$ and $B = 1.85 \pm 0.01$, neighbouring autocovariance (5) (slanting crosses) compared with $-A$ from the previous linear fit, non-overlapping autocovariance (6) (rhombuses) compared with zero, and square root of quadratic autocovariance (7) (crosses) compared with a scaling fit Cm in the range $20 \leq m \leq 50$ with $C = 2.60 \pm 0.03$.

where E_n are the zero mean, uncorrelated identically distributed variables. Independence, again, is not assumed. At variance with the \tilde{R}_n they are independent of calendar lags Δt_n : in fact they are a consequence of price evaluation process, and therefore they are present at each business time.

As a result of the joint action of these two processes one has that after m business lags the observed price evolves according to

$$S_{n+m} = S_n + E_{n+m} - E_n + \sum_{i=0}^{m-1} \tilde{R}_{n+i}. \quad (3)$$

Now one can appreciate that price variation differently depends on two contributions, the first, which represents the evolution of the underlying price, is the sum of the m variables, the second, which represents the uncertainty inherent to the quoting operation, is always the difference of *only two* uncorrelated identically distributed variables.

If this theoretical framework is valid there should be various consequences. First, according to Eq. (3) the variance of a price change after m business time should be

$$\langle (S_{n+m} - S_n)^2 \rangle = 2A + Bm, \quad (4)$$

where $\langle \cdot \rangle$ indicates the average over the probability distribution, and $A = \langle E_n^2 \rangle$ and $B = \langle \tilde{R}_n^2 \rangle$.

This last average also runs on all possible calendar lags between two successive quotations. The occurrence of this property can be seen in Fig. 1 which refers to DEM-USD 1998 exchange quotes, and price variations are given in DEM/10,000.

Results are highly consistent with the linear behaviour with $A = 5.9 \pm 0.2$ and $B = 1.85 \pm 0.01$.

The non-vanishing value of A has an important meaning, since it suggests that the market price models based on continuous time approach [14] are not appropriate: in fact, this would at least require in this case a vanishing price change in the limit of vanishing lag. In this case, price change variance should be zero in the limit $n \rightarrow 0$ (i.e., $A = 0$), while in Fig. 1 it can clearly appreciate the non-occurrence of this fact.

The second, more peculiar, consequence of Eq. (3) is that the neighbouring autocovariance of the two consecutive price variations after m business time is

$$\langle (S_{n+m} - S_n)(S_n - S_{n-m}) \rangle = -A, \quad (5)$$

where A is the same parameter of (4). The similar autocovariance for non-overlapping price variations ($l > 0$), according to Eq. (3), are

$$\langle (S_{n+l+m} - S_{n+l})(S_n - S_{n-m}) \rangle = 0. \quad (6)$$

Indeed, these last two equations represent a strong test in order to verify the goodness of the proposed price modelization, since phenomenology is very peculiar and crudely differs, for example, from Markov anticorrelations.

Both these relations are well satisfied as can be seen in Fig. 1. The same results can be found by using other data sets: $A = 7.7 \pm 0.2$ and $B = 2.59 \pm 0.01$ for DEM-USD 1993 exchange rate, $A = 8.4 \pm 0.4$ and $B = 2.96 \pm 0.03$ for JPY-USD 1993, and $A = 1.2 \pm 0.2$ and $B = 4.88 \pm 0.01$ for JPY-DEM 1993. In the last two cases price variations are given in Yen/100 (pips).

It should be noted that in all cases $2A$ is about 10 times larger than B . This means that the observed price changes are largely dominated by the error effect on a short time scale. Only after about 10 business lags, the underlying price variation is comparable with error and it can be partially appreciated. Ten business lags correspond, approximatively, to 2 min in calendar time for the DEM-USD cases. This means that price change distributions on time scales of few minutes are deeply affected by error, and do not reflect the real price variation.

At this point we are sufficiently convinced that a price is a fuzzy variable, not only because of the obvious bid/ask spread, but especially because the bid price itself cannot be given at each time with an absolute precision less than \sqrt{A} . We call this simple fact the *indeterminacy principle* of markets.

To conclude, let us consider that Eq. (6), which states that the autocovariance of two non-overlapping price variations is zero, may lead to the idea that these variables are independent. The inconsistency of this hypothesis can be provided by computing the following non-overlapping quadratic autocovariance:

$$\langle (S_{n+m+1} - S_{n+1})^2 (S_n - S_{n-m})^2 \rangle - \langle (S_n - S_{n-m})^2 \rangle^2. \quad (7)$$

If two non-overlapping absolute price variations were independent, one simply would have that this quantity vanishes.

On the contrary, the quadratic autocovariance (7) is sensibly different with respect to zero, as it can be appreciated in Fig. 1, where its square root is plotted. In fact,

at large business times m , it shows a linear behaviour Cm . The numerical estimations give $C = 2.60 \pm 0.03$ for DEM-USD 1998 exchange rate, $C = 3.07 \pm 0.08$ for DEM-USD 1993, $C = 2.7 \pm 0.1$ for JPY-USD 1993 and $C = 4.2 \pm 0.3$ for JPY-DEM 1993.

The last result gives an important information concerning the quadratic autocovariance of underlying price process:

$$\langle \tilde{R}_n^2 \tilde{R}_{n+m}^2 \rangle - \langle \tilde{R}_n^2 \rangle^2 \simeq C^2 \quad (8)$$

independent of m at least for business time lag m in the range $20 \leq m \leq 50$ (about 10 min for DEM-USD 1998 exchange rate). In other words, Eq. (8) asserts that absolute price have an autocovariance which is substantially independent of the time separation m . Indeed, this is a very surprising result, and it confirms the presence of a strong correlation on absolute price changes not only on time scales from minutes [15] to months [12,13], but also on shortest time scales of seconds.

Indeed, absolute price exhibits a very rich phenomenology also for what concerns long-term behaviour. In particular, recent results on long memory [12,13] show power-law correlations up to a time scale of one year for $|\Delta S|^x$, where $|\Delta S|$ is price variation and x is a real number, whose exponent depends on x (multiscaling).

3. Single lag distribution and minimum calendar lag distribution

According to (3) the process changes are determined by the joint effect of underlying evolution and added noise. Eq. (3) can be easily rewritten in calendar time as

$$S_{t+\tau} = S_t + E_{n(t+\tau)} - E_{n(t)} + \tilde{R}_{t,\tau}, \quad (9)$$

where t and $t + \tau$ are two calendar times in which quotes are established, and $\tilde{R}_{t,\tau}$ represents the variation of underlying price from time t to time $t + \tau$ corresponding to the sum in (3).

In consequence of (9) and taking into account the uncorrelation of all the variables, the probability distribution of a price change R is given by the convolution

$$P_\tau(R) = Q(\Delta E) \otimes \tilde{P}_\tau(\tilde{R}), \quad (10)$$

where $Q(\Delta E)$ is the error distribution and $\tilde{P}_\tau(\tilde{R})$ is the probability distribution of underlying price \tilde{R} . It should be noted that $Q(\Delta E)$ is directly the probability of the differences of the two variables $E_{n(t+\tau)} - E_{n(t)}$ and not of a single one. The probability $Q(\Delta E)$ does not depend on the calendar time lag τ , at variance with $\tilde{P}_\tau(\tilde{R})$.

The error distribution $Q(\Delta E)$ and the underlying distribution $\tilde{P}_\tau(\tilde{R})$ cannot be directly observed, since the observable probability distribution is only $P_\tau(R)$. Nevertheless, the analysis of the price change variance (4) suggests that at a single business time the distribution is largely ruled by the error distribution. Furthermore, since 2 s is the minimal calendar lag, $(t_{n+1} - t_n)$ is always a multiple of 2 s the corresponding price variations are a subset of those corresponding to the minimal business lag $n = 1$. In the light of these informations, we expected that the minimum calendar time lag distribution P_τ is practically equal to Q .

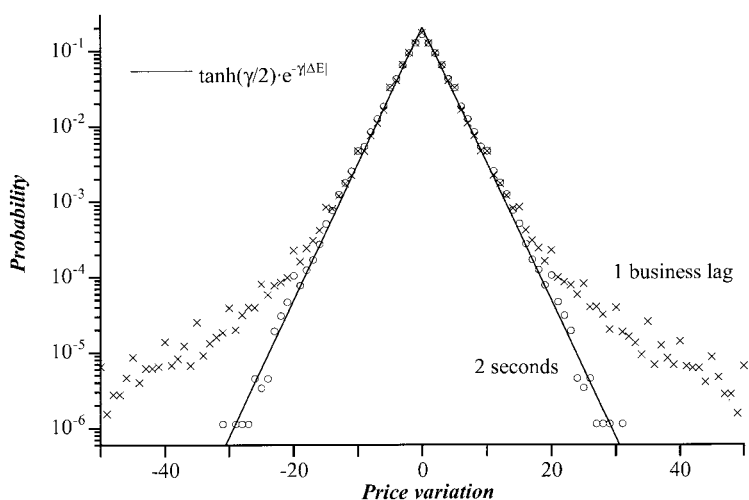


Fig. 2. DEM-USD 1998 exchange rate: price change distribution at minimum calendar lag of 2 s (circles) and at 1 business lag (crosses) in log-linear scales as functions of price variation measured in pips (DEM/10,000). The minimum calendar lag distribution basically coincides with the error distribution $\tanh(\gamma/2) \exp(-\gamma|\Delta E|)$, and this gives the parametric fit $\gamma = 0.40 \pm 0.01$ in the business time range $-23 \leq m \leq +23$.

In Fig. 2 we plot in log-linear scale the 2 s calendar lag distribution. The evidence of Fig. 2 is that the probability distribution $Q(\Delta E)$ is exponential of the form

$$Q(\Delta E) = \tanh \frac{\gamma}{2} \exp(-\gamma|\Delta E|) \quad (11)$$

for six orders of magnitude, and the numerical fit of the parameter gives $\gamma = 0.40 \pm 0.01$ in the business time range $-23 \leq m \leq +23$.

A simple check of self-consistency can be made by computing the variance of the error distribution $Q(\Delta E)$ and comparing the result with $2A$ of Eq. (4). In fact, $Q(\Delta E)$ is the probability distribution of the difference of two independent variables $E_{n(t+\tau)} - E_{n(t)}$, each one having variance equal to A . After some algebra, the variance of distribution (11) reads

$$\langle (\Delta E)^2 \rangle = \frac{1}{\cosh \gamma - 1}. \quad (12)$$

By inserting into this formula the estimated $\gamma = 0.40 \pm 0.02$, one has $\langle (\Delta E)^2 \rangle = 12 \pm 1$, in agreement with the value $2A = 11.8 \pm 0.4$ we have found in the previous section.

In Fig. 2 the minimum business lag distribution is also plotted, for comparison with the 2 s distribution. The central parts of the two distributions are substantially the same for three orders of magnitude, in this region the error process rules. Outside the region the single business lag distribution exhibits more persistent tails. These large events, corresponding to calendar time lags larger than 2 s, are due to underlying price variations instead of erroneous price changes.

4. Distributions at different calendar time lags

In this section we discuss the probability distributions of price variations for calendar lags of length τ from its minimum of 2 s up to time scales of order of few minutes. In particular, we try to retrieve informations about the underlying price distribution $\tilde{P}_\tau(\tilde{R})$ in Eq. (10).

A possible strategy is to guess a functional parametric expression for $\tilde{P}_\tau(\tilde{R})$, and then to compare its $Q(\Delta E)$ -convolution (10) with our experimental probability distributions. In order to make a reasonable hypothesis on $\tilde{P}_\tau(\tilde{R})$, let us come back to business time approach and write down the underlying price change \tilde{R}_n as the product of a volatility σ_n and a Normal Gaussian variable with zero mean and unitary variance W_n :

$$\tilde{R}_n = \sigma_n W_n. \quad (13)$$

Note that this decomposition always allows for independent normal variables W_n . From Eq. (8) and independence of W_n , one can derive a similar relation for the volatility:

$$\langle \sigma_n^2 \sigma_{n+m}^2 \rangle - \langle \sigma_n^2 \rangle^2 \simeq C^2 \quad (14)$$

for m up to 50.

The simplest interpretation of the last result is that volatility remains substantially constant in business time lags of length m , but it assumes different values for temporally far lags (otherwise the last expression should vanish). Then $C^2 \simeq \langle \sigma_n^4 \rangle - \langle \sigma_n^2 \rangle^2$. In other words, in a business lag of m steps the volatility process is substantially frozen, while one has m independent realizations of the Gaussian process. This result identically holds for calendar time, where one says that in a time lag of length τ up to, about, 10 min for DEM-USD 1998 exchange rate, one has a constant volatility. This volatility can obviously change temporally with respect to that of another time lag of length τ .

In other terms, volatility itself is a stochastic process with a characteristic time much larger than about 10 min. The underlying price variation $\tilde{R}_{t,\tau}$ on a calendar lag of length τ is then the sum of several independent Gaussian variables with same variance. Therefore, it can be written as

$$\tilde{R}_{t,\tau} = \sigma_t \sqrt{\tau} W_t, \quad (15)$$

where σ_t is constant in the calendar lag $(t, t + \tau)$ and W_t is a normal Gaussian random variable.

The price process framework is completed when an explicit expression for the volatility probability distribution is given. With this aim, let us recall recent results [5,13], where it was found that returns probability distributions of a stock market daily index and of a foreign daily exchange are symmetric Gaussian distributions whose standard deviations are themselves log-normal processes. Borrowing this result and taking into account the previous considerations, we have the following expression for $\tilde{P}_\tau(\tilde{R})$:

$$\tilde{P}_\tau(\tilde{R}) = \int_0^\infty d\sigma L(\sigma) \int_{\tilde{R}-1/2}^{\tilde{R}+1/2} dr G_{\sigma\sqrt{\tau}}(r), \quad (16)$$

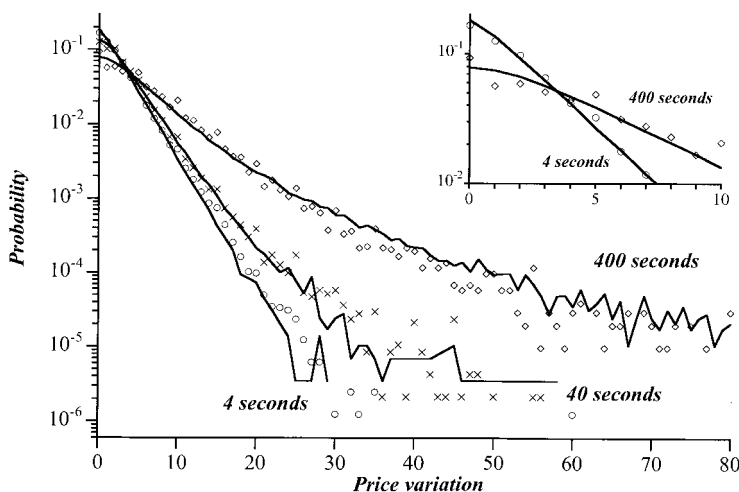


Fig. 3. DEM-USD 1998 exchange rate: price change distributions for calendar time lag τ equal to 4 s (circles), 40 ± 2 s (crosses) and 400 ± 20 s (rhombuses) in log-linear scales as functions of price variation measured in pips (DEM/10,000). The lines represent the numerical computation of the guessed distribution $\tilde{P}_\tau(\tilde{R})$ (16) convoluted with $Q(\Delta E)$ (11) for $\mu = -1.5$ and $\omega = 0.8$. In the inset the small price changes range is magnified for the cases 4 s and 400 s.

where $L(\sigma)$ is a log-normal distributions of parameters μ and ω

$$L(\sigma) = \frac{1}{\omega\sqrt{2\pi}} \exp \left[\frac{-(\ln \sigma - \mu)^2}{2\omega^2} \right] \quad (17)$$

and $G_{\sigma\sqrt{\tau}}(r)$ is a Gaussian distribution with zero mean and standard deviation $\sqrt{\tau}\sigma$, integrated on the unitary interval centered on the integer \tilde{R} .

We have performed a numerical computation of Eq. (16) via a Monte Carlo approach, and then we have convoluted the result with the error distribution $Q(\Delta E)$. For the DEM-USD 1998 exchange rate we have found a parameter choice ($\mu = -1.5$ and $\omega = 0.8$) which gives a good agreement between the guessed and the experimental price change distributions, as shown in Fig. 3, for three values of time lag τ : 4, 40 ± 2 and 400 ± 20 s. In order to have a reasonable number of occurrences, the experimental distributions are computed with a 5% tolerance on τ .

The consistency of the hypothesis of a log-normal volatility which is constant during a whole time lag can be directly checked by computing $\sqrt{\langle \sigma_n^4 \rangle - \langle \sigma_n^2 \rangle^2}$ for such a distribution and comparing the result with the experimental value of C . In the DEM-USD 1998 case one has that the first computation gives $\simeq 2.77$ against the value $C = 2.60 \pm 0.03$. Therefore the constancy of volatility seems reasonably confirmed.

The conclusion is that the underlying volatility itself evolves following a stochastic process, with characteristic times larger than about 10 min, while on shorter time scales the snapshot variance is mainly due to the error process. The hypothesis of a log-normal underlying volatility which seems once more confirmed by experimental data, could be the result of a multiplicative process.

5. Conclusions

We have found evidence that two distinct stochastic processes, an underlying process and an error process, are present in price change dynamics in foreign exchange markets. The latter process, which can be put in connection with the price estimations of market operators, suggests a quantum like nature for price changes variables, in the sense that they have an irreducible intrinsic indeterminacy. For this reason market models based on continuous time limit seem to be not adequate.

Another main consequence of the indeterminacy principle is that price changes cannot be considered as the result of independent stochastic processes, because of the presence of correlations even at the shortest time scales. In fact, the spurious anticorrelations in the observed price changes [16] can be easily explained in the light of the error process.

At this point it seems quite clear that a moving chart over few observed prices is able to give a more accurate estimation of the underlying price at a given time. Nevertheless, the number of observed prices in the moving chart cannot be too large (i.e., calendar time lag cannot be too long) otherwise too large price variations due to the underlying process would decrease accuracy.

We expect to find effects of the indeterminacy principle also in high-frequency market quotes of single stocks. On the contrary, the error component in change distributions of composite market indexes are probably not so important because of the averaging of errors due to the large number of different stocks involved.

Finally, the volatility of the underlying process exhibits a strong autocorrelation, since it turns out to be substantially constant up to time scales of, at least, 10 min. This is a clear evidence that the underlying volatility is a stochastic process with a larger characteristic time, while variance at short time scales is basically due to the error process. We have proposed that the underlying volatility follows from a log-normal process and that the underlying price change distribution is a symmetric Gaussian whose standard deviation is the log-normal stochastic volatility. This hypothesis is in a very good agreement with experimental data.

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