Esercizio Risolvere il seguente problema di PL mediante l'implementazione matriciale del metodo del Simplesso:

$$\begin{aligned} \min\ & -10x_1 - 12x_2 - 12x_3 \\ \text{s.t.}\ & \\ & x_1 + 2x_2 + 2x_3 \leq 20 \\ & 2x_1 + x_2 + 2x_3 \leq 20 \\ & 2x_1 + 2x_2 + x_3 \leq 20 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Soluzione.
Trasformiamo il problema in forma standard:

$$\begin{aligned} \min\ & -10x_1 - 12x_2 - 12x_3 \\ \text{s.t.}\ & \\ & x_1 + 2x_2 + 2x_3 + x_4 = 20 \\ & 2x_1 + x_2 + 2x_3 + x_5 = 20 \\ & 2x_1 + 2x_2 + x_3 + x_6 = 20 \\ & x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \end{aligned}$$

e assumiamo $\mathbf{B} = [\mathbf{A}_4, \mathbf{A}_5, \mathbf{A}_6] = \mathbf{I}$ come base ammissibile iniziale

Iter 1.
$\mathbf{B}^{-1} = \mathbf{I}$

Test_Opt

$\mathbf{u}^T = \mathbf{c}_B^T \mathbf{B}^{-1} = [0 \quad 0 \quad 0]$
$\bar{c}_1 = -10$, $\bar{c}_2 = -12$, $\bar{c}_3 = -12$

$opt = false \implies$ **var. entrante** x_2 [alternativa valida anche x_1]

Test_Illim

$$\bar{\mathbf{A}}_2 = \mathbf{A}_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \implies illim = false$$

calcolo variabile uscente:

$$\bar{\mathbf{b}} = \begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix} \qquad t = \arg\min\{20/2, 20/2, 20/1\} = 1$$

var uscente $x_{B(1)} = x_4$ [alternativa valida anche $B(2) = 5$]

nuova base: $\mathbf{B} = [\mathbf{A}_2, \mathbf{A}_5, \mathbf{A}_6] = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

Iter 2.

$$\mathbf{B}^{-1} \begin{bmatrix} 1/2 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Test_Opt

$\mathbf{u}^T = \mathbf{c}_B^T \mathbf{B}^{-1} = [-6 \quad 0 \quad 0]$

$$\bar{c}_1 = c_1 - \mathbf{u}^T \mathbf{A}_1 = -10 - \begin{bmatrix} -6 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = -4,$$

$$\bar{c}_3 = -\mathbf{u}^T \mathbf{A}_3 = -12 - \begin{bmatrix} -6 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = 0,$$

$$\bar{c}_4 = -\mathbf{u}^T \mathbf{A}_4 = 0 - \begin{bmatrix} -6 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 6$$

$opt = false \implies$ **var. entrante** x_1

Test_Illim

$$\bar{\mathbf{A}}_1 = \begin{bmatrix} 1/2 \\ 3/2 \\ 1 \end{bmatrix} \implies illim = false$$

calcolo variabile uscente:

$$\bar{\mathbf{b}} = \begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix} \qquad t = \arg\min\{10/(1/2), 10/(3/2), 0/1\} = 1$$

var uscente $x_{B(3)} = x_6$ (nota che $\theta = 0$)

nuova base: $\mathbf{B} = [\mathbf{A}_2 \mathbf{A}_5 \mathbf{A}_1] = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 0 & 2 \end{bmatrix}$

Iter 3.

$$\mathbf{B}^{-1} = \begin{bmatrix} 1 & 0 & -1/2 \\ 1 & 1 & -3/2 \\ -1 & 0 & 1 \end{bmatrix}$$

Test_Opt

$$\mathbf{u}^T = \mathbf{c}_B^T \mathbf{B}^{-1} = \begin{bmatrix} -12 & 0 & -10 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1/2 \\ 1 & 1 & -3/2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 & -4 \end{bmatrix}$$

$$\bar{c}_3 = -12 - \begin{bmatrix} -2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = -4,$$

$$\bar{c}_4 = 0 - \begin{bmatrix} -2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 2,$$

$$\bar{c}_6 = 0 - \begin{bmatrix} -2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 4,$$

$opt = false \implies$ **var. entrante** x_3

Test_Illim

$$\bar{\mathbf{A}}_3 = \begin{bmatrix} 0 \\ 5/2 \\ -11 \end{bmatrix} \implies illim = false$$

calcolo variabile uscente:

$$\bar{\mathbf{b}} = \begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix} \qquad t = 2, \theta = \{10/(5/2)\} = 4$$

var uscente $x_{B(2)} = x_5$

nuova base: $\mathbf{B} = [\mathbf{A}_2 \mathbf{A}_3 \mathbf{A}_1] = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

Iter 4.

$$\mathbf{B}^{-1} = \begin{bmatrix} -3/5 & 2/5 & 2/5 \\ 2/5 & -3/5 & 2/5 \\ 2/5 & 2/5 & -3/5 \end{bmatrix}$$

Test_Opt

$$\mathbf{u}^T = \begin{bmatrix} -12 & -12 & -10 \end{bmatrix} \begin{bmatrix} -3/5 & 2/5 & 2/5 \\ 2/5 & -3/5 & 2/5 \\ 2/5 & 2/5 & -3/5 \end{bmatrix} = \begin{bmatrix} -2 & 0 & -4 \end{bmatrix}$$

$$\bar{c}_4 = 0 - \begin{bmatrix} -12/5 & -12/5 & -18/5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 12/5,$$

$$\bar{c}_5 = 0 - \begin{bmatrix} -12/5 & -12/5 & -18/5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 12/5,$$

$$\bar{c}_6 = 0 - \begin{bmatrix} -12/5 & -12/5 & -18/5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 18/5,$$

Test_Opt = true. STOP

Soluzione Ottima $x^* = (4, 4, 4, 0, 0, 0)$