Bouncing, rolling and sticking of stiff and soft bodies

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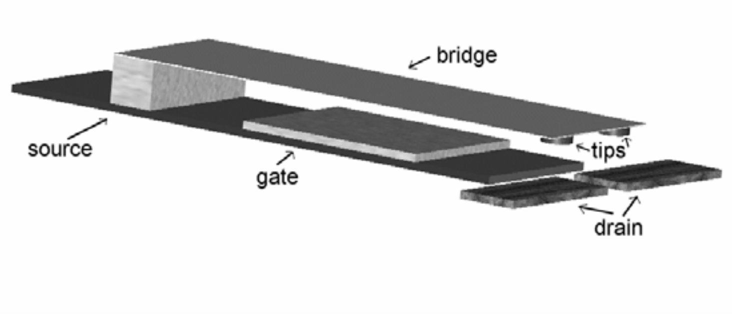
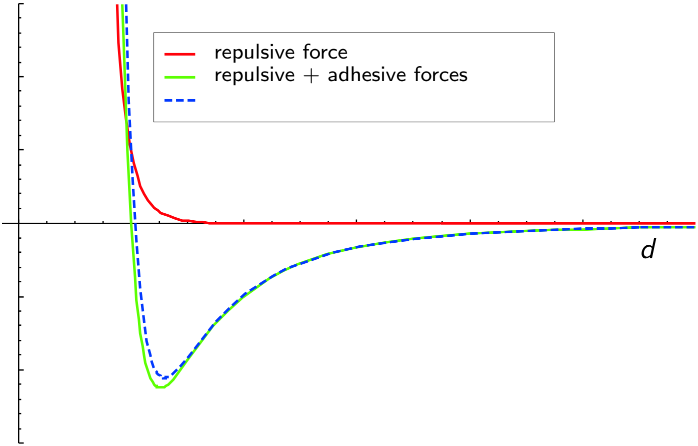
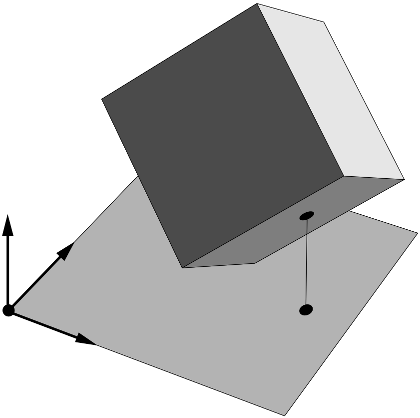
Micro switch




repulsive force
repulsive + adhesive forces
d



Rigid body

A motion of the body $\mathcal{B}$ is described at each time t by a placement defined on the *paragon shape* $\mathcal{D}$:

$$p : \mathcal{D} \times \mathcal{I} \to \mathcal{E}$$

characterized by the following representation:

$$p(x, t) = p_o(t) + R(t)(x - x_o)$$

where $R(t) : \mathcal{V} \to \mathcal{V}$ is a rotation in the translation space of $\mathcal{E}$.

Test velocity fields:

$$w(x) = w_o + WR(t)(x - x_o),$$

with sym $W = 0$

Affine body

A motion of the body $\mathcal{B}$ is described at each time t by a placement defined on the *paragon shape* $\mathcal{D}$:

$$p : \mathcal{D} \times \mathcal{I} \rightarrow \mathcal{E}$$

characterized by the following representation:

$$p(x, t) = p_o(t) + F(t)(x - x_o)$$

where $F(t) : \mathcal{V} \rightarrow \mathcal{V}$ is linear and such that $\det F(t) > 0$

Test velocity fields:

$$w(x) = w_o + GF(t)(x - x_o)$$

Rigid body

Balance principle:

$$\int_{\mathcal{D}} b \cdot w \, dV + \int_{\partial\mathcal{D}} q \cdot w \, dA = 0 \qquad \forall w, \, \forall t$$

Equations of motion:

$$-m \, \ddot{p}_o(t) - m \, g + f(t) = 0$$

$$\mathrm{skw}\big(- \ddot{R}(t) \, J \, R(t)^T + M(t) \, R(t)^T \big) = 0$$

Affine body

Balance principle:

$$-S \cdot GF \operatorname{vol} \mathcal{D} + \int_{\mathcal{D}} b \cdot w \, dV + \int_{\partial \mathcal{D}} q \cdot w \, dA = 0 \qquad \forall w, \, \forall t$$

Equations of motion:

$$-m \, \ddot{p}_o(t) - m \, g + f(t) = 0$$

$$-\ddot{F}(t) \, J \, F(t)^T + \big(M(t) - S(t) \operatorname{vol} \mathcal{D}\big) \, F(t)^T = 0$$

Mass and Euler tensor:

$$m := \int_{\mathcal{D}} \rho \, dV;$$

$$J := \int_{\mathcal{D}} \rho (x - x_o) \otimes (x - x_o) \, dV$$

Total force and moment tensor:

$$f(t) := \int_{\partial \mathcal{D}} q(x, t) \, dA;$$

$$M(t) := \int_{\partial \mathcal{D}} (x - x_o) \otimes q(x, t) dA$$

Piola stress and material properties

Frame indifference:

$$S \cdot WF = 0 \quad \forall W \mid \operatorname{sym} W = 0 \quad \Rightarrow \quad \operatorname{skw} SF^T = 0$$

Mooney-Rivlin strain energy (incompressible material):

$$\varphi(F) := c_{10}(\imath_1(C) - 3) + c_{01}(\imath_2(C) - 3)$$

Stress response (energetic + reactive + dissipative):

$$SF^T = \widehat{S}(F)F^T - \pi\, I + \mu\, \dot{F}F^{-1}$$

$$\widehat{S}(F) \cdot \dot{F} = \frac{d\varphi(F)}{dt} \quad \Rightarrow \quad \widehat{S}(F)\, F^T = 2(c_{10}F\, F^T - c_{01}F^{-T}F^{-1})$$

Dissipation principle:

$$S \cdot \dot{F} - d\varphi(F)/dt \geq 0 \quad \Rightarrow \quad \mu \geq 0$$

Contact force constitutive laws

Repulsive force:

$$q_r(x,t) = \alpha_r \, d(x,t)^{-\nu_r} \, n$$

Damping force:

$$q_d(x,t) = -\beta_d \, d(x,t)^{-\nu_d} \, (n \otimes n) \, \dot{p}(x,t)$$

Frictional force:

$$q_f(x,t) = -\beta_f \, d(x,t)^{-\nu_f} \, (I - n \otimes n) \, \dot{p}(x,t)$$

Adhesive force:

$$q_a(x,t) = -\beta_a \left(d(x,t)^{-\nu_{aa}} - d(x,t)^{-\nu_{ar}} \right) n$$

Contact force constitutive laws

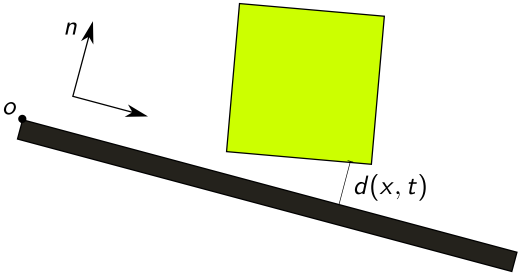
The contact forces are surface forces per unit deformed area:

$$q(x,t) = \sum_j q_j(x,t)\; k(x,t)$$

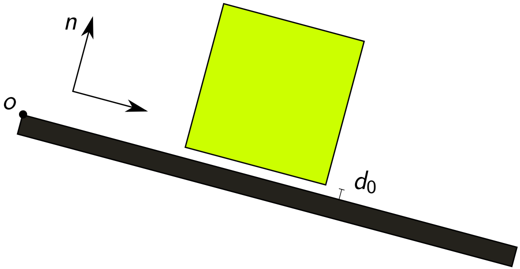
Area change factor:

$$k(x,t) := \|F(t)^{-T} n_{\partial\mathcal{D}}(x)\| \det(F(t))$$

$n_{\partial\mathcal{D}}(x)$ outward unit normal vector



$$d(x,t) := (p(x,t) - o) \cdot n$$



$$d(x, t) := (p(x, t) - o) \cdot n$$

Contact force constitutive laws

Repulsive force:

$$q_r(x,t) = \alpha_r\, d(x,t)^{-\nu_r}\, n$$

Damping force:

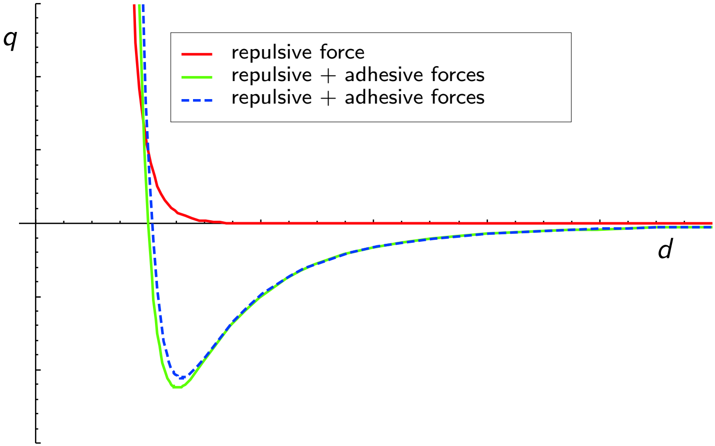
$$q_d(x,t) = -\beta_d\, d(x,t)^{-\nu_d}\, (n \otimes n)\, \dot{p}(x,t)$$

Frictional force:

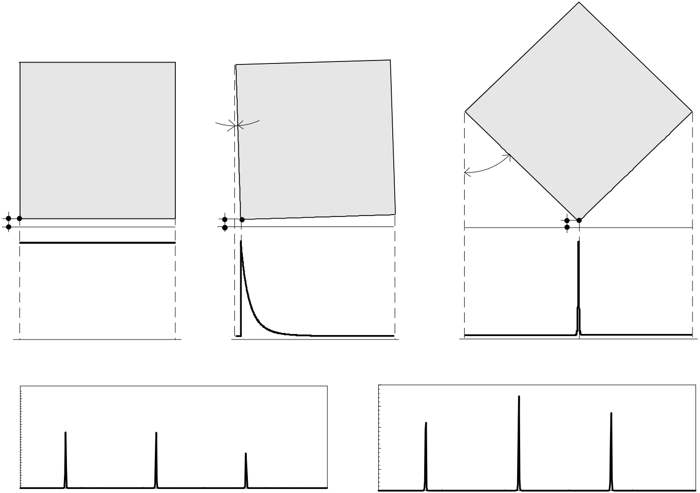
$$q_f(x,t) = -\beta_f\, d(x,t)^{-\nu_f}\, (I - n \otimes n)\, \dot{p}(x,t)$$

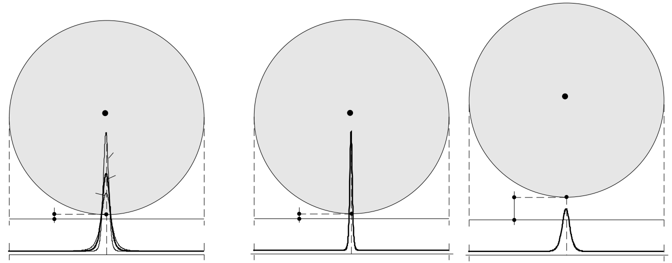
Adhesive force:

$$q_a(x,t) = -\beta_a\, (d(x,t)^{-\nu_{aa}} - d(x,t)^{-\nu_{ar}})\, n$$

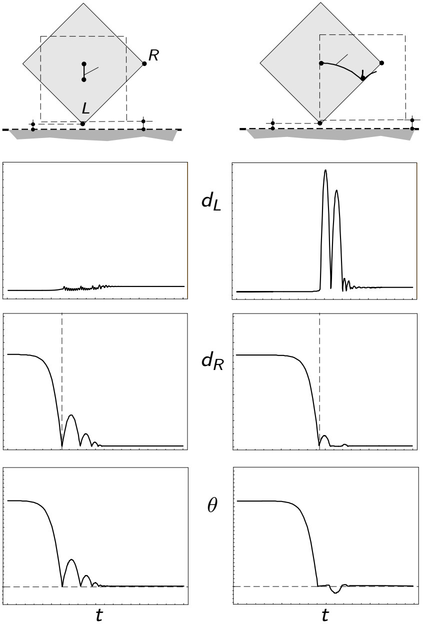


$$\nu_r = 8, \quad \nu_{aa} = 3, \quad \nu_{ar} = 6$$





Numerical simulations



001	011
002	012

Numerical simulations

rocking on a sloping plane

021 022 023

bouncing and rolling

031 032 033 034 035

elastic bouncing and oscillations

041 112
200 214 215 216 217
318 319

adhesion and detachment

501 502 503 505

spinning top

3D-101 3D-111
3D-102 3D-112

dice throwing

3D-201 3D-211
3D-202 3D-212

The end

Appendix

Cauchy stress

$$T = SF^T \frac{1}{\det F}$$

Pressure π

It is the *reactive* part of T. In an incompressible solid/fluid the velocity fields are said to be *isochoric*. The trace of the velocity gradient turns out to be zero.
A reactive stress, whose power is zero for any isochoric velocity field, has to be a spherical tensor $-\pi\, I$:

$$\pi\, I \cdot G = \pi \,\mathrm{tr}\, G = 0$$

Appendix

Mooney-Rivlin

It is a hyperelastic material model used for rubber-like materials as well as for biological tissues.
The *principal invariants* of $C := FF^T$ are defined as

$$\imath_1(C) := F \cdot F, \quad \imath_2(C) := F^\star \cdot F^\star$$

where $F^\star := F^{-T} \det F$ is the *cofactor* of F.

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