$(63-64)_{12}$ Monday [2014-05-12] A1.3
16:00 - 18:00

$$\hat{T}(F)\cdot\dot{F}F^{-1} = \left(\hat{\sigma}_1 - \frac{1}{2}(\hat{\sigma}_2+\hat{\sigma}_3)\right)\frac{\dot{\lambda}}{\lambda}$$

for a neo-Hookean material

$$\frac{d}{dt}\varphi(F) = c_1\left(2\lambda\dot{\lambda} - \frac{2}{\lambda^2}\dot{\lambda}\right) = 2c_1\left(\lambda^2 - \frac{1}{\lambda}\right)\frac{\dot{\lambda}}{\lambda}$$

Setting $\hat{\sigma}_0 := \hat{\sigma}_1 - \frac{1}{2}(\hat{\sigma}_2+\hat{\sigma}_3)$

from $\hat{T}(F)\cdot\dot{F}F^{-1} = \frac{d}{dt}\varphi(F)$

we get $\hat{\sigma}_0 = 2c_1\left(\lambda^2 - \frac{1}{\lambda}\right)$

$$[\operatorname{dev}\hat{T}(F)] = \begin{pmatrix} \hat{\sigma}_1^D & & \\ & \hat{\sigma}_2^D & \\ & & \hat{\sigma}_3^D \end{pmatrix}$$

$$\hat{\sigma}_1^D = \hat{\sigma}_1 - \frac{1}{3}(\hat{\sigma}_1+\hat{\sigma}_2+\hat{\sigma}_3) = \frac{2}{3}\left(\hat{\sigma}_1 - \frac{1}{2}(\hat{\sigma}_2+\hat{\sigma}_3)\right)$$

$$\hat{\sigma}_1^D = \frac{2}{3}\hat{\sigma}_0$$

$$\begin{pmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{pmatrix} = \begin{pmatrix} \hat{\sigma}_1^D & & \\ & \hat{\sigma}_2^D & \\ & & \hat{\sigma}_3^D \end{pmatrix} - p\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

$$T \quad = \quad \operatorname{dev} T \quad + \quad \operatorname{sph} T$$

$$\sigma_1 = \hat{\sigma}_1^D - p \qquad \sigma_1 = \pi$$

$$\sigma_2 = \hat{\sigma}_2^D - p \qquad \sigma_2 = 0$$

$$\sigma_3 = \hat{\sigma}_3^D - p \qquad \sigma_3 = 0$$

$$T = \hat{T}(F) - pI \qquad T = \frac{M}{V_\Omega}$$

material characterization — balance equations

$$\hat{\sigma}_1^D - p = \pi$$

$$\hat{\sigma}_2^D - p = 0$$

$$\hat{\sigma}_3^D - p = 0$$

$$0 - 3p = \pi \quad \text{(taking the trace)}$$

$$\Rightarrow \quad p = -\frac{1}{3}\pi$$

$$\hat{\sigma}_1^D = \pi + p = \frac{2}{3}\pi$$

$$\hat{\sigma}_2^D = p = -\frac{1}{3}\pi$$

$$\hat{\sigma}_3^D = p = -\frac{1}{3}\pi$$

$$\hat{\sigma}_1^D = \frac{2}{3}\hat{\sigma}_0 \quad \Rightarrow \quad \frac{2}{3}\hat{\sigma}_0 = \frac{2}{3}\pi$$

$$\boxed{2c_1\left(\lambda^2 - \frac{1}{\lambda}\right) = \pi}$$

$(65\text{-}66)_{12}$ Tuesday [2014-05-13] A1.3

9:00 - 11:00

$$T = \hat{T}(F) - pI + T^{+} \qquad T^{+} \cdot \dot{F}F^{-1} \geq 0$$

dissipation principle

The standard choice for a viscous material is

$$T^{+} = 2\mu \,\mathrm{sym}\, \dot{F}F^{-1}$$

with $\mu \geq 0$ by the dissipation principle.

$$[T^{+}] = \begin{pmatrix} \sigma_1^{+} & & \\ & \sigma_2^{+} & \\ & & \sigma_3^{+} \end{pmatrix} = 2\mu \begin{pmatrix} 1 & & \\ & -\frac{1}{2} & \\ & & -\frac{1}{2} \end{pmatrix} \frac{\dot{\lambda}}{\lambda}$$

$$\sigma_1 = \hat{\sigma}_1^{D} - p + \sigma_1^{+} \qquad \sigma_1 = n$$

$$\sigma_2 = \hat{\sigma}_2^{D} - p + \sigma_2^{+} \qquad \sigma_2 = 0$$

$$\sigma_3 = \hat{\sigma}_3^{D} - p + \sigma_3^{+} \qquad \sigma_3 = 0$$

material characterization — balance equations

$$\hat{\sigma}_1^{D} - p + \sigma_1^{+} = n$$

$$\hat{\sigma}_2^{D} - p + \sigma_2^{+} = 0$$

$$\hat{\sigma}_3^{D} - p + \sigma_3^{+} = 0$$

$$0 - 3p + 0 = n \quad \Rightarrow \quad p = -\frac{1}{3}n$$

$$\hat{\sigma}_1^D - p + \sigma_1^+ = p$$

$$\frac{2}{3}\hat{\sigma}_0 + \sigma_1^+ = p + p = \frac{2}{3}p$$

$$\hat{\sigma}_0 + \frac{3}{2}\sigma_1^+ = p$$

$$\boxed{2c_1\left(\lambda^2 - \frac{1}{\lambda}\right) + 3\mu\frac{\dot{\lambda}}{\lambda} = p}$$

Find a solution for a given value p_0.

Let λ_0 be such that

$$2c_1\left(\lambda_0^2 - \frac{1}{\lambda_0}\right) = p_0$$

and let us look for a solution close to λ_0

$$\lambda(t) = \lambda_0 + \tilde{\varepsilon}(t) \quad \text{with } \tilde{\varepsilon} \text{ a small quantity } \forall t$$

$$= 1 + \underbrace{\varepsilon_0 + \tilde{\varepsilon}(t)}$$

$$= 1 + \varepsilon(t)$$

$$\Rightarrow \quad \dot{\lambda}(t) = \dot{\tilde{\varepsilon}}(t) = \dot{\varepsilon}(t)$$

We assume the motion is so slow that even $\dot{\tilde{\varepsilon}}$ is a small quantity $\forall t$.

$$\lambda^2 = (\lambda_0 + \tilde{\varepsilon})^2 = \lambda_0^2 + 2\lambda_0\tilde{\varepsilon} + o(\tilde{\varepsilon})$$

$$\frac{1}{\lambda} = \frac{1}{\lambda_0} - \frac{1}{\lambda_0^2}\tilde{\varepsilon} + o(\tilde{\varepsilon})$$

$$\frac{\dot{\lambda}}{\lambda} = \left(\frac{1}{\lambda_0} - \frac{1}{\lambda_0^2}\tilde{\varepsilon}\right)\dot{\tilde{\varepsilon}} + o(\tilde{\varepsilon}) = \frac{1}{\lambda_0}\dot{\tilde{\varepsilon}} + o(\tilde{\varepsilon})$$

$$2c_1\left(\lambda_0^2 + 2\lambda_0\tilde{\varepsilon} - \frac{1}{\lambda_0} + \frac{1}{\lambda_0^2}\tilde{\varepsilon}\right) + 3\mu\frac{1}{\lambda_0}\dot{\tilde{\varepsilon}} = p_0$$

$$\underbrace{2c_1\left(\lambda_0^2 - \frac{1}{\lambda_0}\right)}_{p_0} + 2c_1\left(2\lambda_0 + \frac{1}{\lambda_0^2}\right)\tilde{\varepsilon} + 3\mu\frac{1}{\lambda_0}\dot{\tilde{\varepsilon}} = p_0$$

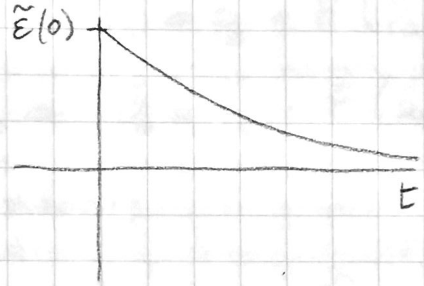
$$2c_1\left(2\lambda_0^2 + \frac{1}{\lambda_0}\right)\tilde{\varepsilon} + 3\mu\dot{\tilde{\varepsilon}} = 0$$

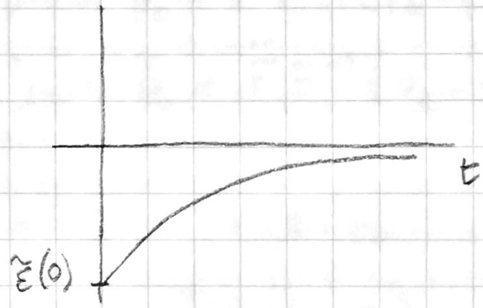
$(67-68)_{12}$ Wednesday [2014-05-14] A1.3
9:00-11:00

$$a\,\tilde{\varepsilon}(t) + \dot{\tilde{\varepsilon}}(t) = 0$$

$$a := \frac{2\,c_1}{3\,\mu}\left(2\,\lambda_0^2 + \frac{1}{\lambda_0}\right)$$

$$\tilde{\varepsilon}(t) = \tilde{\varepsilon}(0)\,e^{-a t}$$



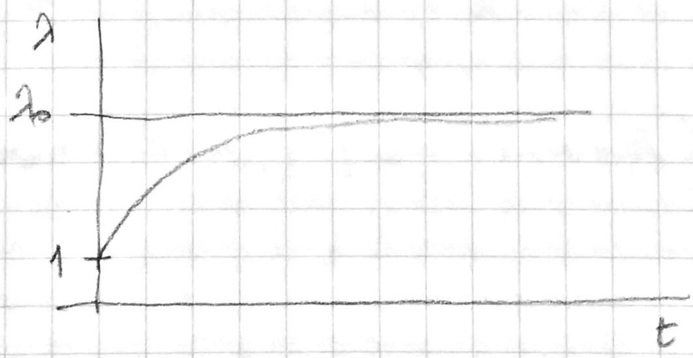


If ε_0 is very small we can set

$$\tilde{\varepsilon}(0) = -\,\varepsilon_0$$

$$\Rightarrow \quad \lambda(0) = 1 + \varepsilon_0 + \tilde{\varepsilon}(0) = 1$$

$$\lambda(t) = 1 + \varepsilon_0\left(1 - e^{-a t}\right)$$



The viscosity makes the deformation evolve in time. If the traction p is applied suddenly at time $t = 0$ the body starts deforming and will approach the final shape with a decreasing velocity as $t \to \infty$.

