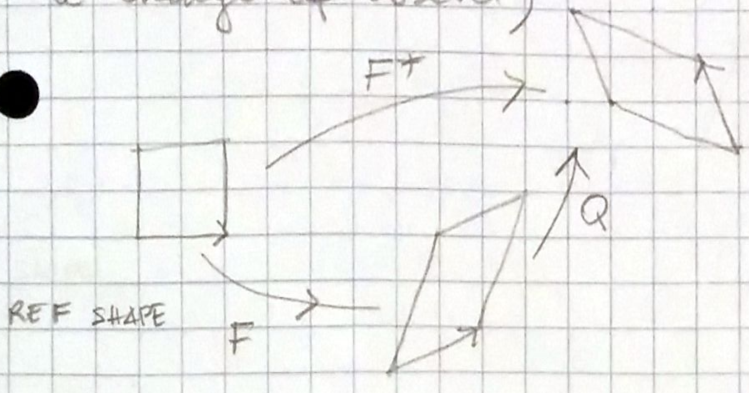
[2016-04-21]

$$S \cdot \dot{F} = \frac{d}{dt}\varphi(F)$$

strain energy

Objectivity (frame indifference, invariance under a change of observer)



$$\varphi(F) = \varphi(F^*)$$

$$\Rightarrow \quad \varphi(F) = \varphi(QF) \qquad \forall Q, \ \forall F$$

$$Q = R^T \Rightarrow \varphi(F) = \varphi(QRU) = \varphi(U)$$

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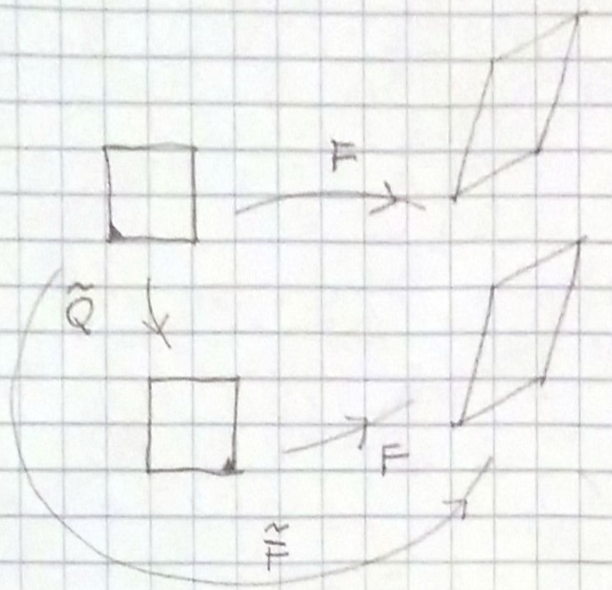
Material Symmetry



$$\tilde{\tilde F} = F Q$$

$\tilde Q$ symmetry $\qquad \varphi(F\tilde Q) = \varphi(F) \qquad$ symmetry invariance

$$\downarrow \qquad\qquad \downarrow \text{obj}$$

$$\varphi(RU\tilde Q) = \varphi(U)$$

$$\downarrow$$

$$\varphi(R\tilde Q\,\underbrace{\tilde Q^T U \tilde Q})$$

$$\text{obj}\downarrow$$

$$\varphi(\tilde Q^T U \tilde Q) = \varphi(U)$$

$\hat T(F\tilde Q) = \hat T(F) \qquad$ symmetry invariance

$$\hat T(RU\tilde Q) = \hat T(F)$$

$$\hat T(\underbrace{R\tilde Q}\,\underbrace{\tilde Q^T U \tilde Q}) = \hat T(F)$$

$$\text{obj} \downarrow \qquad\qquad \searrow \text{obj}$$

$$R\tilde Q\,\hat T(\tilde Q^T U \tilde Q)\,\tilde Q^T R^T = R\,\hat T(U)\,R^T$$

$$\tilde Q\,\hat T(\tilde Q^T U \tilde Q)\,\tilde Q^T = \hat T(U)$$

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[2016-04-26]

Isotropic hyperelastic material

$$\varphi(\mathsf{F}) = \hat{\varphi}(\iota_1, \iota_2, \iota_3)$$

$$\hat{\mathsf{S}}_e(\mathsf{F}) = 2\mathsf{F}\Big((\varphi_{,1} + \varphi_{,2}\,\iota_1)\mathsf{I} - \varphi_{,2}\,\mathsf{C} + \varphi_{,3}\,\iota_3\,\mathsf{C}^{-1}\Big)$$

$$\hat{\mathsf{T}}_e(\mathsf{F}) = \frac{2}{\sqrt{\iota_3}}\Big((\varphi_{,1} + \varphi_{,2}\,\iota_1)\mathsf{B} - \varphi_{,2}\,\mathsf{B}^2 + \varphi_{,3}\,\iota_3\,\mathsf{I}\Big)$$

$\mathsf{B} = \mathsf{F}\mathsf{F}^T$ left Cauchy-Green tensor

$$\frac{d}{dt}\iota_1 = \frac{d}{dt}(\mathsf{F}\cdot\mathsf{F}) = 2\,\mathsf{F}\cdot\dot{\mathsf{F}}$$

$$\frac{d}{dt}\iota_2 = \frac{d}{dt}\frac{1}{2}\Big((\mathrm{tr}\,\mathsf{C})^2 - \mathrm{tr}\,\mathsf{C}^2\Big) = \frac{d}{dt}\frac{1}{2}\big(\iota_1^2 - \mathsf{C}\cdot\mathsf{C}\big)$$

$$= 2\,\iota_1\,\mathsf{F}\cdot\dot{\mathsf{F}} - \mathsf{C}\cdot\dot{\mathsf{C}} = 2\,\iota_1\,\mathsf{F}\cdot\dot{\mathsf{F}} - \mathsf{C}\cdot(\dot{\mathsf{F}}^T\mathsf{F} + \mathsf{F}^T\dot{\mathsf{F}})$$

$$= 2\,\iota_1\,\mathsf{F}\cdot\dot{\mathsf{F}} - 2\,\mathsf{C}\cdot\mathsf{F}^T\dot{\mathsf{F}} = (2\,\iota_1\,\mathsf{F} - 2\,\mathsf{F}\mathsf{C})\cdot\dot{\mathsf{F}}$$

$$\frac{d}{dt}\iota_3 = \frac{d}{dt}(\det\mathsf{F})^2 = 2(\det\mathsf{F})\frac{d}{dt}(\det\mathsf{F})$$

$$= 2(\det\mathsf{F})(\det\mathsf{F})\,\mathrm{tr}(\dot{\mathsf{F}}\mathsf{F}^{-1}) = 2\,\iota_3\,\mathsf{F}^{-T}\cdot\dot{\mathsf{F}}$$

$$= 2\,\iota_3\,\mathsf{F}\mathsf{C}^{-1}\cdot\dot{\mathsf{F}}$$

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Isochoric motion

$$\det F(t) = 1$$

$$\frac{d}{dt}(\det F) = (\det F)\, tr\,(\dot F F^{-1}) = \overset{1}{(\det F)}\, tr\, \nabla v = div\, v = 0$$

Velocity gradient decomposition

$$\nabla v = \underbrace{\nabla v - \frac{1}{3}(tr\, \nabla v) I}_{deviatoric} \quad + \underbrace{\frac{1}{3}(tr\, \nabla v) I}_{spherical}$$

Deformation gradient decomposition

$$F = F_v F_I \qquad\qquad F_v = (\det F)^{1/3} I$$

$$\dot F F^{-1} = (\dot F_v F_I + F_v \dot F_I) F_I^{-1} F_v^{-1}$$

$$= \dot F_v F_v^{-1} + F_v (\dot F_I F_I^{-1}) F_v^{-1}$$

$$= \dot F_v F_v^{-1} + \dot F_I F_I^{-1}$$

$$tr\, \dot F_I F_I^{-1} = \left(\frac{d}{dt}(\det F_I)\right)\frac{1}{\det F_I} = 0$$

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Decomposition of the stretching

$$\mathbf{D} := \operatorname{sym} \nabla v$$

$$\mathbf{L} = \mathbf{W} + \mathbf{D}$$

$$\mathbf{T} \cdot \mathbf{L} = \mathbf{T} \cdot \mathbf{W} + \mathbf{T} \cdot \mathbf{D} = \mathbf{T} \cdot \mathbf{D}$$

$$\mathbf{L} = \operatorname{sph} \mathbf{L} + \operatorname{dev} \mathbf{L} = \operatorname{sph} \mathbf{D} + \operatorname{dev} \mathbf{D} + \mathbf{W}$$

$$\operatorname{sph} \mathbf{L} = \left(\frac{1}{3} \operatorname{tr} \mathbf{L}\right) \mathbf{I} \quad \Rightarrow \quad \operatorname{dev} \mathbf{D} = \mathbf{D} - \frac{1}{3} (\operatorname{tr} \mathbf{D}) \mathbf{I}$$

$$\operatorname{tr}(\operatorname{dev} \mathbf{D}) = \operatorname{tr} \mathbf{D} - \frac{1}{3} (\operatorname{tr} \mathbf{D})\, 3 = 0$$

$$\nabla v = \dot{\mathbf{F}} \mathbf{F}^{-1}$$

$$\mathbf{F} = (\det \mathbf{F})^{1/3} \underbrace{\mathbf{F} (\det \mathbf{F})^{-1/3}}_{\mathbf{F}_I}$$

$$\det\left(\mathbf{F} (\det \mathbf{F})^{-1/3}\right) = \det \mathbf{F} \, \frac{1}{\det \mathbf{F}} = 1$$

$$\dot{\mathbf{F}}_I = -\frac{1}{3} (\det \mathbf{F})^{-\frac{4}{3}} (\det \mathbf{F}) \operatorname{tr}(\dot{\mathbf{F}} \mathbf{F}^{-1}) \mathbf{F} + (\det \mathbf{F})^{-1/3} \dot{\mathbf{F}}$$

$$= (\det \mathbf{F})^{-\frac{1}{3}} \left(-\frac{1}{3} \operatorname{tr}(\dot{\mathbf{F}} \mathbf{F}^{-1}) \mathbf{F} + \dot{\mathbf{F}} \right)$$

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$$\dot{F}_I F_I^{-1} = \dot{F} F^{-1} - \frac{1}{3} \operatorname{tr}(\dot{F} F^{-1}) I$$

$$\begin{aligned}
\dot{F} F^{-1} &= (F_V F_I)^{\dot{}} (F_V F_I)^{-1} \\
&= (\dot{F}_V F_I + F_V \dot{F}_I)(F_I^{-1} F_V^{-1}) \\
&= \dot{F}_V F_V^{-1} + F_V \dot{F}_I F_I^{-1} F_V^{-1} \\
&= \dot{F}_V F_V^{-1} + \dot{F}_I F_I^{-1}
\end{aligned}$$

$$\begin{aligned}
\dot{F}_V F_V^{-1} &= \frac{1}{3} (\det F)^{-\frac{2}{3}} (\det F) \operatorname{tr}(\dot{F} F^{-1}) (\det F)^{-\frac{1}{3}} I \\
&= \frac{1}{3} \operatorname{tr}(\dot{F} F^{-1}) I
\end{aligned}$$

$$\Rightarrow \quad \dot{F}_I F_I^{-1} = \operatorname{dev}(\dot{F} F^{-1})$$

$$\dot{F}_V F_V^{-1} = \operatorname{sph}(\dot{F} F^{-1})$$

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