AN INTRODUCTION TO CONTINUUM MECHANICS

Let $\mathcal{E}$ be the set of positions taken by a small body (a particle), together with a vector space $\mathcal{V}$, closed under the operation "translation"

$$+ : \mathcal{E} \times \mathcal{V} \to \mathcal{E}$$

$$x + u = y \qquad x, y \in \mathcal{E},\ u \in \mathcal{V}$$

transforming a position x into a position y with the following properties

i) for any ordered couple (x, y) there is only one vector, denoted by $y - x$ translating x to y

ii) for any position x and any two vectors u and v

$$(x + u) + v = x + (u + v)$$

iii) for any position x

$$x + o = x$$

where o is the null vector of $\mathcal{V}$

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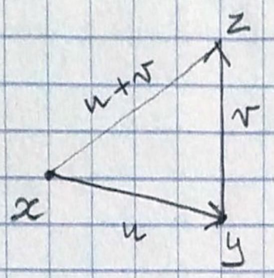
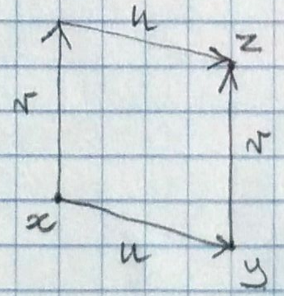
If we use dots and arrows for describing positions and vectors we can describe the second property by drawing a triangle



Since $(x+u)+v = (x+v)+u$

we can describe this equality by drawing a parallelogram



The set of positions $\mathcal{E}$ just defined is called the Euclidean Space

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Distance between two positions x, y

$$d(x,y) \geq 0 \qquad d(x,y) = d(y,x)$$

$$d(x,y) = 0 \iff x = y$$

$$d(x,y) = \| y - x \|$$

$$\| u \| = (u \cdot u)^{1/2}$$

with $\quad u \cdot v \in \mathbb{R} \qquad \forall u, v \in V$

$$(\alpha u) \cdot v = \alpha (u \cdot v)$$

$$v \cdot u = u \cdot v$$

$$u \cdot u \geq 0 \qquad u \cdot u = 0 \iff u = 0$$

orthogonality $\quad u \cdot v = 0$

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body $\mathcal{B}$ collection of particles

Collective "behaviour" description

motion

$$p : \mathcal{B} \times \mathcal{I} \to \mathcal{E}$$

time interval $\mathcal{I} \subset \mathbb{R}$

placement at time t

$p_t \qquad p : \mathcal{B} \times \{t\} \to \mathcal{E}$

shape of body $\mathcal{B}$ at time t

$$\mathcal{R} = \operatorname{im} p_t \qquad \mathcal{R} \subset \mathcal{E}$$

trajectory of particle A

$p_A \qquad p : \{A\} \times \mathcal{I} \to \mathcal{E}$

deformation

$$\phi : \bar{\mathcal{R}} \to \mathcal{R}$$

reference shape

current shape

reference placement

$$\bar{p} : \mathcal{B} \to \bar{\mathcal{R}} \subset \mathcal{E}$$

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