[2018-03-21]

$\kappa : \mathcal{D} \to \bar{\mathcal{R}} \subset \mathcal{E}$ parameterization of $\bar{\mathcal{R}}$

$$
\begin{tikzcd}
 & \bar{\mathcal{R}} \arrow[r, "\phi"] & \mathcal{R} \\
\mathcal{D} \arrow[ur, "\kappa"] & \mathcal{B} \arrow[u, "\bar{p}"'] &
\end{tikzcd}
$$

$\phi_\kappa := \phi \circ \kappa : \mathcal{D} \to \mathcal{R}$ parameterization of $\mathcal{R}$

Description of ϕ through κ

$$\phi_\kappa(s_1, s_2, s_3) = \phi(\kappa(s_1, s_2, s_3))$$

$$
\begin{aligned}
\phi_\kappa(s_1, s_2, s_3) = 0 &+ \phi_{\kappa 1}(s_1, s_2, s_3)\, e_1 \\
&+ \phi_{\kappa 2}(s_1, s_2, s_3)\, e_2 \\
&+ \phi_{\kappa 3}(s_1, s_2, s_3)\, e_3
\end{aligned}
$$

Let us chose κ such that

$$\kappa(s_1, s_2, s_3) = 0 + s_1 e_1 + s_2 e_2 + s_3 e_3$$

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$$\bar{c}_1(h) = \bar{p}_0 + h\, e_1 = \kappa(s_1, s_2, s_3) + h\, e_1$$

$$= o + s_1 e_1 + s_2 e_2 + s_3 e_3 + h\, e_1$$

$$= \kappa(s_1 + h, s_2, s_3)$$

$$c_1(h) = \phi(\bar{c}_1(h)) = \phi(\kappa(s_1+h, s_2, s_3))$$

$$= \phi_\kappa(s_1+h, s_2, s_3) = o + \phi_{\kappa 1}(s_1+h, s_2, s_3)\, e_1$$

$$+ \phi_{\kappa 2}(s_1+h, s_2, s_3)\, e_2 + \phi_{\kappa 3}(s_1+h, s_2, s_3)\, e_3$$

$$c_1'(0) = \lim_{h\to 0} (c_1(h) - c_1(0))\frac{1}{h}$$

$$= \partial_1 \phi_{\kappa 1}\, e_1 + \partial_1 \phi_{\kappa 2}\, e_2 + \partial_1 \phi_{\kappa 3}\, e_3$$

$$c_2(h) = \phi_\kappa(s_1, s_2+h, s_3) = \ldots$$

$$c_3(h) = \phi_\kappa(s_1, s_2, s_3+h) = \ldots$$

$$\bar{c}(h) = \bar{p}_0 + h\,\overbrace{(\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3)}^{\bar{u}}$$

$$= \kappa(s_1 + h\alpha_1, s_2 + h\alpha_2, s_3 + h\alpha_3)$$

$$c(h) = \phi_\kappa(s_1 + h\alpha_1, s_2 + h\alpha_2, s_3 + h\alpha_3)$$

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$$\bar{c}'(0) = \lim_{h \to 0} \left(\bar{c}(h) - \bar{c}(0) \right) \frac{1}{h}$$

$$c'(0) = \lim_{h \to 0} \left(c(h) - c(0) \right) \frac{1}{h}$$

$$\bar{c}(h) = o + (s_1 + \alpha_1 h)\, e_1 + (s_2 + \alpha_2 h)\, e_2 + (s_3 + \alpha_3 h)\, e_3$$

$$\begin{aligned} c(h) = o &+ \phi_{\kappa 1}(s_1 + \alpha_1 h,\ s_2 + \alpha_2 h,\ s_3 + \alpha_3 h)\, e_1 \\ &+ \phi_{\kappa 2}(\qquad\qquad)\, e_2 \\ &+ \phi_{\kappa 3}(\qquad\qquad)\, e_3 \end{aligned}$$

$$\begin{aligned} c'(0) = {} & \partial_1 \phi_{\kappa 1} \alpha_1 e_1 + \partial_2 \phi_{\kappa 1} \alpha_2 e_1 + \partial_3 \phi_{\kappa 1} \alpha_3 e_1 \\ & + \partial_1 \phi_{\kappa 2} \alpha_1 e_2 + \partial_2 \phi_{\kappa 2} \alpha_2 e_2 + \partial_3 \phi_{\kappa 2} \alpha_3 e_2 \\ & + \partial_1 \phi_{\kappa 3} \alpha_1 e_3 + \partial_2 \phi_{\kappa 3} \alpha_2 e_3 + \partial_3 \phi_{\kappa 3} \alpha_3 e_3 \end{aligned}$$

$$\bar{c}_1'(0) \mapsto c_1'(0) = \partial_1 \phi_{\kappa 1}\, e_1 + \partial_1 \phi_{\kappa 2}\, e_2 + \partial_1 \phi_{\kappa 3}\, e_3$$

$$\bar{c}_2'(0) \mapsto c_2'(0) = \partial_2 \phi_{\kappa 1}\, e_1 + \partial_2 \phi_{\kappa 2}\, e_2 + \partial_2 \phi_{\kappa 3}\, e_3$$

$$\bar{c}_3'(0) \mapsto c_3'(0) = \partial_3 \phi_{\kappa 1}\, e_1 + \partial_3 \phi_{\kappa 2}\, e_2 + \partial_3 \phi_{\kappa 3}\, e_3$$

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$$\bar{c}'(0) \mapsto c'(0) = \alpha_1 c_1'(0) + \alpha_2 c_2'(0) + \alpha_3 c_3'(0)$$

$\Rightarrow$ there exists a tensor $F(\bar{p}_0)$ such that

$$F(\bar{p}_0)\left(\alpha_1 \bar{c}_1' + \alpha_2 \bar{c}_2' + \alpha_3 \bar{c}_3'\right) = \alpha_1 c_1' + \alpha_2 c_2' + \alpha_3 c_3'$$

$$c_1' = F\bar{c}_1' \,, \quad c_2' = F\bar{c}_2' \,, \quad c_3' = F\bar{c}_3'$$

$$[F] = \begin{pmatrix} \phi_{k1,1} & \phi_{k1,2} & \phi_{k1,3} \\ \phi_{k2,1} & \phi_{k2,2} & \phi_{k2,3} \\ \phi_{k3,1} & \phi_{k3,2} & \phi_{k3,3} \end{pmatrix}.$$

We call F the DEFORMATION GRADIENT.

Let $\quad o(h) := c(h) - \left(c(0) + F(\bar{p}_0)\left(\bar{c}(h) - \bar{c}(0)\right)\right)$

then

$$\lim_{h \to 0} o(h) = 0$$

$$\lim_{h\to 0} \frac{o(h)}{h} = \lim_{h \to 0} \frac{c(h) - c(0)}{h} - F(\bar{p}_0) \lim \frac{\bar{c}(h) - \bar{c}(0)}{h}$$

$$= c'(0) - F(\bar{p}_0)\,\bar{c}'(0) = 0$$

$$\Rightarrow \quad \lim_{h \to 0} \frac{o(h)}{h} = 0$$

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