Rigid motion velocity gradient

$F = R$

$\dot{F}F^{-1} = \underbrace{\dot{R}R^T}_{W}$ spin tensor

$RR^T = I \Rightarrow \underbrace{\dot{R}R^T}_{W} + \underbrace{R\dot{R}^T}_{W^T} = 0$

skewsymmetry $W + W^T = 0$

$Wa = \alpha a \Rightarrow Wa \cdot a = \alpha\, a \cdot a$

$\downarrow$

$a \cdot W^T a$

$\downarrow$

$-a \cdot Wa \Rightarrow -\alpha\, a \cdot a = \alpha\, a \cdot a \Rightarrow \alpha = 0$

$$[W] = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \qquad \omega := \omega_1 e_1 + \omega_2 e_2 + \omega_3 e_3$$

$\uparrow \; We_1 \quad \uparrow \; We_2 \quad \uparrow \; We_3$

$$W\omega = \omega_1 We_1 + \omega_2 We_2 + \omega_3 We_3$$
$$= (\omega_1 \omega_3 - \omega_3 \omega_1)\, e_2 + \dots \qquad = 0$$

$$v(p_A) = v(p_o) + W(p_A - p_o)$$

$$w_A := v(p_A) - (v(p_A)\cdot\omega)\frac{\omega}{\omega\cdot\omega} \quad \Rightarrow \quad w_A \cdot \omega = 0$$

$$w_o := v(p_o) - (v(p_o)\cdot\omega)\frac{\omega}{\omega\cdot\omega} \quad \Rightarrow \quad w_o \cdot \omega = 0$$

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$$v(p_A)\cdot\omega = v(p_0)\cdot\omega + \underbrace{\cancel{W(p_A-p_0)\cdot\omega}}_{(p_A-p_0)\cdot W^T\omega = -(p_A-p_0)\cdot W\omega = 0}$$

$$w_A + \cancel{\underbrace{(v(p_A)\cdot\omega)}}\frac{\omega}{\omega\cdot\omega} = w_0 + \cancel{\underbrace{(v(p_0)\cdot\omega)}}\frac{\omega}{\omega\cdot\omega} + W(p_A-p_0)$$

$$=$$

From $v(p_0)$ and $W$ we can compute $\omega$ and $w_0$.

Is there a position $p_A$ such that $w_A = 0$ ?

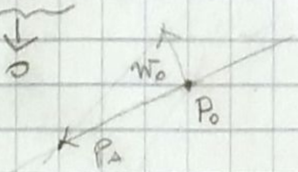
$$w_A = w_0 + Wr \qquad\qquad r = p_A - p_0$$

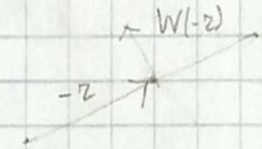
$$\Downarrow \qquad\qquad W(-r) = w_0$$

$$0 = w_0\cdot r + \underbrace{Wr\cdot r}_{\Downarrow\ 0}$$

$$\Downarrow$$

$$0 = w_0\cdot r$$




$$Wr = (-\omega_3 r_2 + \omega_2 r_3)\,e_1$$
$$+ (\omega_3 r_1 - \omega_1 r_3)\,e_2$$
$$+ (-\omega_2 r_1 + \omega_1 r_1)\,e_3$$

Let $r\cdot\omega = 0 \qquad r_1\omega_1 + r_2\omega_2 + r_3\omega_3 = 0$

$$\Downarrow$$

if $\omega_3 \neq 0 \qquad r_3 = -r_1\dfrac{\omega_1}{\omega_3} - r_2\dfrac{\omega_2}{\omega_3}$

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$$\begin{cases} \omega_3 r_2 - \omega_2 r_3 = w_{01} \\ \omega_1 r_3 - \omega_3 r_1 = w_{02} \\ \omega_2 r_1 - \omega_1 r_2 = w_{03} \end{cases}$$

$$\begin{cases} \omega_3 r_2 + r_1 \left( \dfrac{\omega_1}{\omega_3} \omega_2 \right) + r_2 \left( \dfrac{\omega_2}{\omega_3} \omega_2 \right) = w_{01} \\[2ex] -\omega_3 r_1 - r_1 \left( \dfrac{\omega_1}{\omega_3} \omega_1 \right) - r_2 \left( \dfrac{\omega_2}{\omega_3} \omega_1 \right) = w_{02} \end{cases}$$

$$\begin{cases} r_2 \left( \omega_3 + \dfrac{\omega_2^2}{\omega_3} \right) + r_1 \dfrac{\omega_1 \omega_2}{\omega_3} = w_{01} \\[2ex] r_2 \dfrac{\omega_2 \omega_1}{\omega_3} + r_1 \left( \omega_3 + \dfrac{\omega_1^2}{\omega_3} \right) = -w_{02} \end{cases}$$

$$\begin{cases} r_2 \dfrac{\omega_3^2 + \omega_2^2}{\omega_3} + r_1 \dfrac{\omega_1 \omega_2}{\omega_3} = w_{01} \\[2ex] r_2 \dfrac{\omega_2 \omega_1}{\omega_3} + r_1 \dfrac{\omega_3^2 + \omega_1^2}{\omega_3} = -w_{02} \end{cases}$$

$$\omega_1 = 0 \,,\; \omega_2 = 0 \;\Rightarrow\; \begin{cases} r_2 = w_{01} \dfrac{1}{\omega_3} \\[2ex] r_1 = -w_{02} \dfrac{1}{\omega_3} \end{cases}$$

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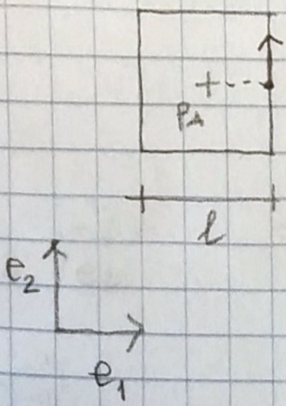
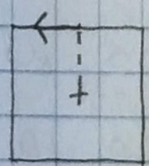



$$(x - p_A) = \frac{\ell}{2}\, e_1$$

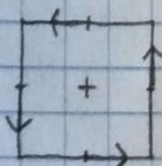
$$W(x - p_A) = \frac{\ell}{2}\, W e_1 = \frac{\ell}{2}\, \omega_3\, e_2$$

$$e_3 \in \operatorname{span}\{\omega\} \Rightarrow [W] = \begin{pmatrix} 0 & -\omega_3 & 0 \\ \omega_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$$(x - p_A) = \frac{\ell}{2}\, e_2$$

$$W(x - p_A) = \frac{\ell}{2}\, W e_2 = -\frac{\ell}{2}\, \omega_3\, e_1$$



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