Affine motion velocity gradient

$F = RU$

$$\nabla v = \dot{F}F^{-1} = (\dot{R}U + R\dot{U})(U^{-1}R^T)$$

$$= \dot{R}R^T + R\dot{U}U^{-1}R^T$$

$$U = \lambda_1 P_1 + \lambda_2 P_2 + \lambda_3 P_3$$
$$= \lambda_1 (a_1 \otimes a_1) + \lambda_2 (a_2 \otimes a_2) + \lambda_3 (a_3 \otimes a_3)$$

$a_1(t) = Q(t)\, e_1$

$a_2(t) = Q(t)\, e_2 \qquad Q(t)^T Q(t) = I$

$a_3(t) = Q(t)\, e_3$

$$\dot{a}_1 = \dot{Q}\, e_1 = \dot{Q} Q^T a_1$$

$$\dot{U} = \dot{\lambda}_1 P_1 + \dot{\lambda}_2 P_2 + \dot{\lambda}_3 P_3$$
$$+ \lambda_1 (\dot{Q} e_1 \otimes Q e_1 + Q e_1 \otimes \dot{Q} e_1)$$
$$+ \lambda_2 (\qquad\qquad)$$
$$+ \lambda_3 (\qquad\qquad)$$

[Notebook page scanned on 2018/04/05]

Special case

$$\begin{cases} Q = I & \alpha = 0 \\ R = I & \omega = 0 \end{cases}$$

$$F = U$$

$$\dot{F} F^{-1} = \dot{U} U^{-1}$$

$$U(t) = \lambda_1(t)\, P_1 + \lambda_2(t)\, P_2 + \lambda_3(t)\, P_3$$

$$P_1 = e_1 \otimes e_1 \,, \quad P_2 = e_2 \otimes e_2 \,, \quad P_3 = e_3 \otimes e_3$$

Let us set

$$\lambda_1(t) = 1 + \varepsilon \cos(\Omega t) \qquad |\varepsilon| < 1$$

$$\lambda_2(t) = 1 + \varepsilon \sin(\Omega t)$$

$$\lambda_3(t) = 1$$

$$\dot{U} U^{-1} = \frac{\dot{\lambda}_1}{\lambda_1} P_1 + \frac{\dot{\lambda}_2}{\lambda_2} P_2 \qquad \begin{aligned} \dot{\lambda}_1 &= -\varepsilon\, \Omega \sin \Omega t \\ \dot{\lambda}_2 &= \varepsilon\, \Omega \cos \Omega t \end{aligned}$$

[Notebook page scanned on 2018/04/05]

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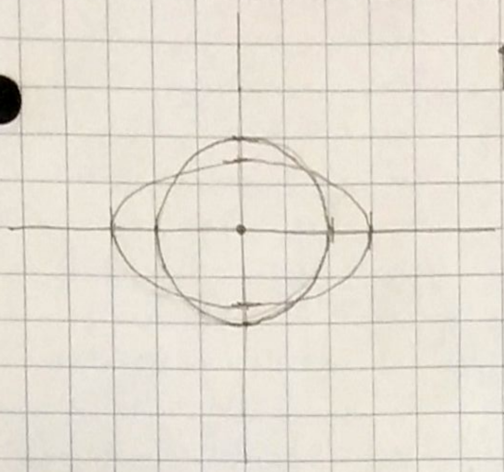
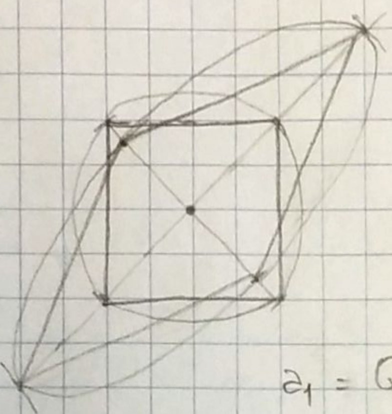


$$F = U = \lambda_1 P_1 + \lambda_2 P_2$$

$$P_1 = e_1 \otimes e_1$$

$$P_2 = e_2 \otimes e_2$$

$$\nabla v = \dot{U} U^{-1} = \frac{\dot{\lambda}_1}{\lambda_1}(e_1 \otimes e_1) + \frac{\dot{\lambda}_2}{\lambda_2}(e_2 \otimes e_2)$$



For a more general stretch

$$P_1 = a_1 \otimes a_1$$

$$P_2 = a_2 \otimes a_2$$

$$a_1 = Q e_1 = \cos\gamma \, e_1 + \sin\gamma \, e_2$$

$$a_2 = Q e_2 = -\sin\gamma \, e_1 + \cos\gamma \, e_2$$

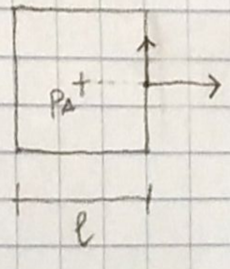
$$P_1 = Q e_1 \otimes Q e_1 = Q(e_1 \otimes e_1) Q^T$$

$$P_2 = Q e_2 \otimes Q e_2 = Q(e_2 \otimes e_2) Q^T$$

$$\nabla v = \dot{U} U^{-1} = \frac{\dot{\lambda}_1}{\lambda_1} Q(e_1 \otimes e_1) Q^T + \frac{\dot{\lambda}_2}{\lambda_2} Q(e_2 \otimes e_2) Q^T$$

$$[P_1] = \begin{pmatrix} \cos^2\gamma & \cos\gamma \sin\gamma & 0 \\ \sin\gamma \cos\gamma & \sin^2\gamma & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad [P_2] = \begin{pmatrix} \sin^2\gamma & -\sin\gamma \cos\gamma & 0 \\ -\cos\gamma \sin\gamma & \cos^2\gamma & 0 \\ 0 & & 0 \end{pmatrix}$$

[Notebook page scanned on 2018/04/05]



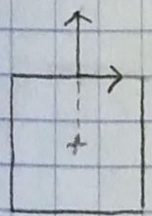
$$(x - p_A) = \frac{\ell}{2} e_1$$

$$\nabla v \, (x - p_A) = \frac{\ell}{2} \nabla v \, e_1$$

$$= \frac{\ell}{2} \frac{\dot\lambda_1}{\lambda_1} P_1 e_1 + \frac{\ell}{2} \frac{\dot\lambda_2}{\lambda_2} P_2 e_1$$

$$= \frac{\ell}{2} \frac{\dot\lambda_1}{\lambda_1} \cos^2\gamma \, e_1 + \frac{\ell}{2} \frac{\dot\lambda_1}{\lambda_1} \sin\gamma \cos\gamma \, e_2$$

$$+ \frac{\ell}{2} \frac{\dot\lambda_2}{\lambda_2} \sin^2\gamma \, e_1 - \frac{\ell}{2} \frac{\dot\lambda_2}{\lambda_2} \cos\gamma \sin\gamma \, e_2$$



$$(x - p_A) = \frac{\ell}{2} e_2$$

$$\nabla v \, (x - p_A) = \frac{\ell}{2} \nabla v \, e_2$$

$$= \frac{\ell}{2} \frac{\dot\lambda_1}{\lambda_1} P_1 e_2 + \frac{\ell}{2} \frac{\dot\lambda_2}{\lambda_2} P_2 e_2$$

$$= \frac{\ell}{2} \frac{\dot\lambda_1}{\lambda_1} \cos\gamma \sin\gamma \, e_1 + \frac{\ell}{2} \frac{\dot\lambda_1}{\lambda_1} \sin^2\gamma \, e_2$$

$$- \frac{\ell}{2} \frac{\dot\lambda_2}{\lambda_2} \sin\gamma \cos\gamma \, e_1 + \frac{\ell}{2} \frac{\dot\lambda_2}{\lambda_2} \cos^2\gamma \, e_2$$

[Notebook page scanned on 2018/04/05]