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Symmetric and skew-symmetric tensors

$$\operatorname{sym} A := \frac{1}{2}\left(A + A^T\right)$$

$$\operatorname{skw} A := \frac{1}{2}\left(A - A^T\right)$$

$$\Rightarrow \quad \operatorname{sym} A + \operatorname{skw} A = A$$

Since

$$A \cdot B = A^T \cdot B^T$$

for a skew-symmetric tensor W we get

$$A \cdot W = A^T \cdot W^T$$

$$A \cdot W = -A^T W$$

$$\underbrace{(A + A^T)}_{\text{symmetric}} \cdot W = 0$$

skew-symmetric

Hence the scalar product of a symmetric tensor by a skew-symmetric tensor is 0.

As a consequence

$$A \cdot B = (\operatorname{sym} A + \operatorname{skw} A) \cdot (\operatorname{sym} B + \operatorname{skw} B)$$

$$= (\operatorname{sym} A) \cdot (\operatorname{sym} B) + (\operatorname{skw} A) \cdot (\operatorname{skw} B)$$

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More properties about scalar products of Tensors

$$A \cdot BC = tr\left(A(BC)^T\right) = tr\left(AC^T B^T\right) = AC^T \cdot B$$

$$A \cdot DB = A^T \cdot B^T D^T = tr\left(A^T D B\right) = D^T A \cdot B$$

$$A \cdot I = tr(A)$$

Cross product

For any skew-symmetric tensor W we get

$$f \cdot Wu = (f \otimes u) \cdot W = \frac{1}{2}(f \otimes u - u \otimes f) \cdot W$$

Denoting by w the axial vector of W, by cross product between u and f we mean a vector

$$m = u \times f$$

such that $\quad m \cdot w = \frac{1}{2}\left(f \otimes u - u \otimes f\right) \cdot W \quad \forall W$

Using an orthonormal basis $\{e_1, e_2, e_3\}$ and tensors

$W_1 = -e_2 \otimes e_3 + e_3 \otimes e_2 \qquad$ (axial vector e_1)

$W_2 = -e_3 \otimes e_1 + e_1 \otimes e_3 \qquad$ (axial vector e_2)

$W_3 = -e_1 \otimes e_2 + e_2 \otimes e_3 \qquad$ (axial vector e_3) $\quad$ we get

$m_1 = m \cdot e_1 = \frac{1}{2}(f \otimes u - u \otimes f) \cdot W_1 = u_2 f_3 - u_3 f_2 \quad [\ldots]$

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