Force distributions

By force distribution $\mathcal{F}$ applied to a body $\mathcal{B}$ in its current shape $\mathcal{R}$ we mean a linear real function over the vector space of the test velocity fields;

we assume that for any two test velocity fields v_1 and v_2, the vector field $(v_1 + v_2)$ such that

$$(v_1 + v_2)(x) = v_1(x) + v_2(x)$$

is again a test velocity field, as it is the vector field αv such that

$$(\alpha v)(x) = \alpha(x)\, v(x)$$

for any scalar field α and any test velocity field v.

The test velocity fields v are assumed to be continous on $\mathcal{R}$ and differentiable with continuous gradient tensor fields ∇v.

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The force distribution $\mathfrak{F}$ is usually given the following representation

$$\mathfrak{F}(v) = \int_{\mathcal{R}} b(x) \cdot v(x)\, dV + \int_{\partial\mathcal{R}} t(x) \cdot v(x)\, dA \qquad \forall v$$

where b is called the bulk force density per unit current volume and t is called just the traction, which is a force density per unit current area.

We can supplement the expression above with a singular distribution made up of terms like

$$f_A \cdot v(p_A)$$

where f_A is called a force applied to the body point A taking the position $p_A \in \mathcal{R}$.

The real value $\mathfrak{F}(v)$ is called the (total) power, while $b(x) \cdot v(x)$ or $t(x) \cdot v(x)$ are called power density at x per unit current volume or per unit current area respectively.

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Velocity gradient and force distributions

- rigid test velocity field gradient

$$\nabla v = \underbrace{\dot{R} R^T}_{W} \qquad W^T = - W$$

- affine test velocity field gradient

$$\nabla v = \dot{F} F^{-1}$$

$$L = W + D$$

W: skew symmetric part; D: symmetric part

Force distribution $\mathcal{F}(v)$

general representation in continuum mechanics

$$\mathcal{F}(v) = \int b(x) \cdot v(x)\, dV + \int t(x) \cdot v(x)\, dA$$

Singular (concentrated) force distribution

$$\mathcal{F}(v) = f_A \cdot v(p_A) + f_B \cdot v(p_B) + f_C \cdot v(p_C) + \cdots$$

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For any affine (possibly rigid) test velocity field

$$v(x) = v_0 + L(x - p_0)$$

a singular force distribution can be given the following representation

$$\mathcal{F}(v) = f_A \cdot v(p_A) + f_B \cdot v(p_B)$$

$$= f_A \cdot v_0 + f_A \cdot L(p_A - p_0) + f_B \cdot v_0 + f_B \cdot L(p_B - p_0)$$

$$= f_A \cdot v_0 + f_A \otimes (p_A - p_0) \cdot L + f_B \cdot v_0 + f_B \otimes (p_B - p_0) \cdot L$$

$$= (f_A + f_B) \cdot v_0 + \big(f_A \otimes (p_A - p_0) + f_B \otimes (p_B - p_0)\big) \cdot L$$

$$\mathcal{F}(v) = \underset{\substack{\uparrow \\ \text{total force}}}{f} \cdot v_0 + \underset{\substack{\uparrow \\ \text{total moment tensor}}}{M} \cdot L$$

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